20 points Math 220 Homework 4 Sections 3.1-3.5 18 problems

Most of these problems are from Jiri Lebl's "Notes on Diffy Q's, Differential Equations for Engineers."

1. Find the general solution to $\begin{cases} x' = 3x \\ y' = x + y \\ z' = x + z \end{cases}$ without using the eigenvalue method. Then rewrite the system in $\vec{u}'(t) = A\vec{u}(t)$ form and use the eigenvalue method to get the same answer.

2. Find the general solution to $\begin{cases} x_1' = 2tx_2 \\ x_2' = 2tx_2 \end{cases}$ without using the eigenvalue method. Then rewrite the system in $\vec{u}'(t) = A\vec{u}(t)$ form and use the eigenvalue method to get the same answer.

3. Are $\begin{bmatrix} e^t \\ e^{2t} \end{bmatrix}$ and $\begin{bmatrix} e^{2t} \\ e^t \end{bmatrix}$ linearly independent? Justify.

4. Are $\begin{bmatrix} e^t \\ 1 \end{bmatrix}$, $\begin{bmatrix} \cosh(t) \\ 1 \end{bmatrix}$, and $\begin{bmatrix} e^{-t} \\ 1 \end{bmatrix}$ linearly independent? Justify.

5. Find the general solution to $\vec{u}'(t) = A\vec{u}(t)$ if $A = \begin{bmatrix} 1 & 0 & 3 \\ -1 & 0 & 1 \\ 2 & 0 & 2 \end{bmatrix}$.

6. Find the general solution to $\begin{cases} x' = x - 2y \\ y' = 4x + 5y \end{cases}$. Is (0,0) a stable (sink) or unstable (source) equilibrium solution (critical point)? Is it a node, spiral, or saddle? If there is rotation, is it CW or CCW? Defend your answers.

7. Find the general solution for $x' = y$, $y' = x$ using the eigenvalue method.

8. Find the general solution for $x' = y$, $y' = -x$ using the eigenvalue method.

9. Solve $\vec{u}' = \begin{bmatrix} 0.5 & 0 \\ 1 & -0.5 \end{bmatrix}$ if $\vec{u}(0) = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$.

10. Find the general solution to $\begin{cases} x' = 2x + 5y \\ y' = -x + 6y \end{cases}$. (0,0) is an unstable spiral. In what direction do the spirals rotate? Defend your answer.

11. Is (0,0) a stable (sink) or unstable (source) equilibrium solution (critical point)? Is it a node, spiral, center, saddle or degenerate node? Defend your answers using eigenvalues or trace and determinant.
a) $x_1' = x_1 + x_2$, $x_2' = 2x_2$
b) $x' = -2y$, $y' = 2x$
c) $x_1' = x_1 + 3x_2$, $x_2' = -2x_1 - 4x_2$
d) $x' = x - 4y$, $y' = -4x + y$

The general solution to $\vec{u}'(t) = A\vec{u}(t)$ is given in the following problems. Graph a representative set of solutions and classify the critical point (0,0). Which of your representative solutions best estimates the solution if $\vec{u}(0) = (1,1)$?

12. $\vec{u} = C_1 e^{-t} \begin{bmatrix} 2 \\ -1 \end{bmatrix} + C_2 e^{-6t} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

13. $e^{3t}\left(C_1 \begin{bmatrix} 2\cos(2t) - \sin(2t) \\ 3\cos(2t) + 4\sin(2t) \end{bmatrix} + C_2 \begin{bmatrix} \cos(2t) + 2\sin(2t) \\ -4\cos(2t) + 3\sin(2t) \end{bmatrix}\right).$

    What eigenvector and eigenvalue were used to construct this solution?

14. $\vec{u} = C_1 e^{-t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + C_2 e^{-t} \left(t \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0.2 \end{bmatrix} \right)$

15. $\vec{u} = C_1 e^{-t} \begin{bmatrix} 2 \\ -1 \end{bmatrix} + C_2 e^{3t} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

16. Find the general solution to $\vec{u}'(t) = A\vec{u}(t)$ if $A = \begin{bmatrix} 2 & -1 & -2 \\ 0 & 1 & -2 \\ 0 & 2 & 1 \end{bmatrix}$.

17. Find the general solution to $\vec{u}'(t) = A\vec{u}(t)$ if $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

18. CAS problem (3 points): use a CAS device for the following. Turn in a pdf file as specified in the syllabus. **Matlab notes for this lab follow the brief answers.**

    Solve $\begin{cases} \dfrac{dx}{dt} = -3x - 4y + \cos(t) \\ \dfrac{dy}{dt} = x - y - \sin(t) \end{cases}$ if $\begin{bmatrix} x(0) \\ y(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$. Graph $x(t)$ and $y(t)$ on the same set of axes with $0 \le t \le 40$. Then draw the "phase plane" for the solution for $0 \le t \le 40$. the phase plane is the plot of $(x(t), y(t))$ in the xy-plane. Explain briefly how the two graphs correspond.

Brief answers

1. $x = 2C_1e^{3t}, y = C_2e^t + C_1e^{3t}, z = C_3e^t + C_1e^{3t}$ vs. $\vec{u} = C_1e^{3t} \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} + C_2e^t \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + C_3e^t \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

2. $x_1 = C_1 + C_2e^{t^2}$ and $x_2 = C_2e^{t^2}$ vs. $\vec{u} = C_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + C_2e^{t^2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

3. Yes.

4. No.

5. $\vec{u} = C_1 e^{4t} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + C_2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + C_3 e^{-t} \begin{bmatrix} 3 \\ 5 \\ -2 \end{bmatrix}$

6. $e^{3t} \left(C_1 \begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} \cos(2t) \\ \sin(2t) \end{bmatrix} + C_1 \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \cos(2t) \\ \sin(2t) \end{bmatrix} \right)$. $(0,0)$ is an unstable spiral rotating CCW.

7. $\vec{u} = C_1 e^{t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + C_2 e^{-t} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

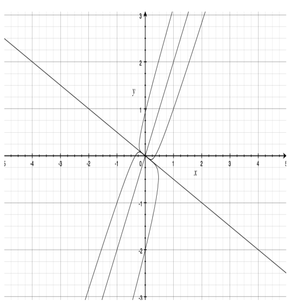
8. $\vec{u} = C_1 \begin{bmatrix} \cos(t) \\ -\sin(t) \end{bmatrix} + C_2 \begin{bmatrix} \sin(t) \\ \cos(t) \end{bmatrix}$.

9. $\vec{u} = 3e^{t/2} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + 2e^{-t/2} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

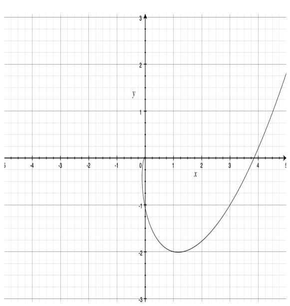
10. $e^{4t} \left(C_1 \begin{bmatrix} 5\cos(t) \\ 2\cos(t) - \sin(t) \end{bmatrix} + C_2 \begin{bmatrix} 5\sin(t) \\ \cos(t) + 2\sin(t) \end{bmatrix} \right)$ and the spirals rotate CW.

11. a) unstable node (or source node) b) stable center (or semistable center) c) stable node (or node sink). d) Unstable saddle

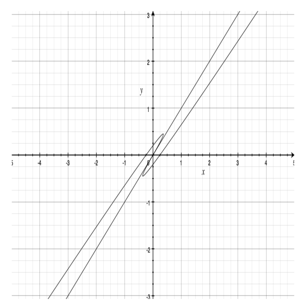
12. Stable node, or sink.



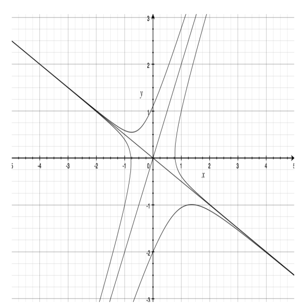
13. Unstable CCW spiral.
$\lambda = 3 + 2i$; $\begin{bmatrix} 2+i \\ 3-4i \end{bmatrix}$



14. Degenerate stable node.



15. Unstable saddle.



16. $\vec{u}(t) = C_1 e^{2t} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + e^{t} \left(C_2 \begin{bmatrix} \cos(2t) \\ \cos(2t) \\ \sin(2t) \end{bmatrix} + C_3 \begin{bmatrix} \sin(2t) \\ \sin(2t) \\ -\cos(2t) \end{bmatrix} \right)$

17. $\vec{u} = C_1 e^{3t} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + C_2 \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + C_3 \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$

MatLab Notes

Notes for Homework #4

```
syms x(t) y(t)
eqns=[diff(x,t)==-2*x+2*y, diff(y,t)==x-3*y];
conds=[x(0)==1,y(0)==2];
[xSol(t) ySol(t)]=dsolve(eqns,conds)
```

xSol(t) = $2\,e^{-t} - e^{-4\,t}$

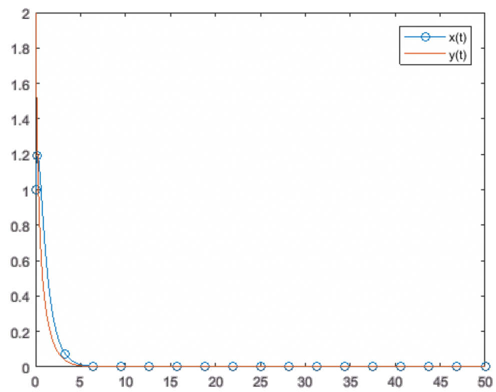
ySol(t) = $e^{-t} + e^{-4\,t}$

```
fplot(xSol(t),[0 50],'o-')
```

Warning: MATLAB has disabled some advanced graphics rendering features by switching to

software OpenGL. For more information, click here.

```
hold on
fplot(ySol(t),[0 50])
legend('x(t)', 'y(t)')
hold off
```




```
fplot(xSol(t),ySol(t),[0 50])
title('Phase Plane')
```