20 points Math 220 Homework 5 Sections 3.7-3.9 18 problems

Most of these problems are from Jiri Lebl's "Notes on Diffy Q's, Differential Equations for Engineers."

1. Is $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & -4 \\ 0 & 1 & 6 \end{bmatrix}$ defective or complete? Defend your answer.

2. Is $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 3 \end{bmatrix}$ defective or complete? Defend your answer.

3. If A is a complete 3 x 3 matrix with an eigenvalue of 2 three times, what matrix must A be? Prove your answer is correct.

4. Find the general solution to $\begin{cases} x' = 5x + 4y \\ y' = -x + y \end{cases}$.

5. Find the general solution to $\begin{cases} x' = -4x - y \\ y' = x - 2y \end{cases}$.

6. Find the general solution to $\vec{u}'(t) = A\vec{u}(t)$ if $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 1 & 0 \\ -1 & 1 & 2 \end{bmatrix}$.

7. Find the general solution to $\vec{u}'(t) = A\vec{u}(t)$ if $A = \begin{bmatrix} 2 & 0 & 0 \\ -1 & -1 & 9 \\ 0 & -1 & 5 \end{bmatrix}$.

8. $A = \begin{bmatrix} a & a \\ b & c \end{bmatrix}$ has an eigenvalue of 5 with multiplicity 2 and eigenspace of $C \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. Find A and show that there is only one solution.

9. Compute e^{At} and then use it to solve $\vec{u}' = \begin{bmatrix} 3 & -1 \\ 1 & 1 \end{bmatrix} \vec{u}$ if $\vec{u}(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

10. Compute e^{At} if $A = \begin{bmatrix} 2 & 0 & 2 \\ 0 & 1 & 0 \\ 2 & 0 & 2 \end{bmatrix}$.

11. Compute e^{At} if $A = \begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix}$.

12. Compute the first three terms of the Taylor series expansion of e^{At} where $A = \begin{bmatrix} 2 & 3 \\ 2 & 2 \end{bmatrix}$. Write your answer as a single matrix and then use it to approximate $e^{0.1A}$.

13. Find the general solution for $\begin{cases} \dfrac{dx}{dt} = 2x + 3y - 7 \\ \dfrac{dy}{dt} = -x - 2y + 5 \end{cases}$.

14. Find the general solution for $\frac{d\vec{u}}{dt} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & 5 \end{bmatrix} \vec{u} + e^{4t} \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$.

15. Find the general solution for $\frac{d\vec{u}}{dt} = \begin{bmatrix} 0 & 2 \\ -1 & 3 \end{bmatrix} \vec{u} + e^{t} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

16. Find the general solution for $\frac{d\vec{u}}{dt} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \vec{u} + \begin{bmatrix} \sec(t) \\ 0 \end{bmatrix}$.

17. Find the general solution for $\frac{d\vec{u}}{dt} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \vec{u} + e^{t} \begin{bmatrix} \cos(t) \\ \sin(t) \end{bmatrix}$.

18. CAS problem (3 points): use a CAS device for the following. Turn in a pdf file as specified in the syllabus. **Matlab notes for this lab follow the brief answers.**

    Use **ode45** to estimate a solution to $\begin{cases} \frac{dx}{dt} = x + y^2 - t^3 \\ \frac{dy}{dt} = x^3 + y + \cos(t) \end{cases}$ if $\begin{bmatrix} x(1) \\ y(1) \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ for $-2 \leq t \leq 1$. do not print the values of $x(t)$ and $y(t)$ computed by ode45; instead, draw the graphs of $x(t)$ and $y(t)$ on the same set of axes with horizontal t-axis. Be sure to mark them differently and provide a legend. Notice that the initial value is given at $t = 1$, so the step size will have to be negative. Also note the outputs explode at -2, so your t-interval input for ode45 must be put in backwards as $[1 - 1.9]$. If that is done, the initial value will automatically be used for $t = 1$, and you will not be returned an error for estimates made outside the tolerance levels.

Brief answers

1. Complete.

2. Defective

3. $A = 2I$.

4. $\vec{u} = C_1 e^{3t} \begin{bmatrix} 2 \\ -1 \end{bmatrix} + C_2 e^{3t} \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 2 \\ -1 \end{bmatrix} \right)$.

5. $\vec{u} = C_1 e^{-3t} \begin{bmatrix} 1 \\ -1 \end{bmatrix} + C_2 e^{-3t} \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right)$.

6. $\vec{u} = C_1 e^{t} \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} + C_2 e^{t} \left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \right) + C_3 e^{2t} \begin{bmatrix} 3 \\ 3 \\ -2 \end{bmatrix}$

7. $\vec{u} = C_1 e^{2t} \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix} + C_2 e^{2t} \left(\begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix} \right) + C_3 e^{2t} \left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} + \frac{t^2}{2} \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix} \right)$

8. $A = \begin{bmatrix} 5 & 5 \\ 0 & 5 \end{bmatrix}$

9. $e^{At} = e^{2t} \begin{bmatrix} t+1 & -t \\ t & 1-t \end{bmatrix}$ and $\vec{u}(t) = e^{2t} \begin{bmatrix} 1-t \\ 2-t \end{bmatrix}$.

10. $e^{At} = \dfrac{1}{2} \begin{bmatrix} e^{4t}+1 & 0 & e^{4t}-1 \\ 0 & 2e^{t} & 0 \\ e^{4t}-1 & 0 & e^{4t}+1 \end{bmatrix}$

11. $e^{At} = \dfrac{1}{2} \begin{bmatrix} e^{3t}+e^{-t} & -e^{3t}+e^{-t} \\ -e^{3t}+e^{-t} & e^{3t}+e^{-t} \end{bmatrix}$

12. $e^{At} \approx \begin{bmatrix} 1+2t+5t^2 & 3t+6t^2 \\ 2t+4t^2 & 1+2t+5t^2 \end{bmatrix}$ and so $e^{0.1A} \approx \begin{bmatrix} 1.25 & 0.36 \\ 0.24 & 1.25 \end{bmatrix}$.

13. $\vec{u} = C_1 e^{-t} \begin{bmatrix} -1 \\ 1 \end{bmatrix} + C_2 e^{t} \begin{bmatrix} -3 \\ 1 \end{bmatrix} + \begin{bmatrix} -1 \\ 3 \end{bmatrix}$.

14. $\vec{u} = C_1 e^{t} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + C_2 e^{2t} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + C_3 e^{5t} \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} - e^{4t} \begin{bmatrix} 1.5 \\ 3.5 \\ 2 \end{bmatrix}$

15. $\vec{u} = C_1 e^{t} \begin{bmatrix} 2 \\ 1 \end{bmatrix} + C_2 e^{2t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + e^{t} \begin{bmatrix} 3 \\ 3 \end{bmatrix} + te^{t} \begin{bmatrix} 4 \\ 2 \end{bmatrix}$.

16. $\vec{u} = C_1 \begin{bmatrix} \cos(t) \\ \sin(t) \end{bmatrix} + C_2 \begin{bmatrix} \sin(t) \\ -\cos(t) \end{bmatrix} + t \begin{bmatrix} \cos(t) \\ \sin(t) \end{bmatrix} + \ln|\cos(t)| \begin{bmatrix} -\sin(t) \\ \cos(t) \end{bmatrix}$.

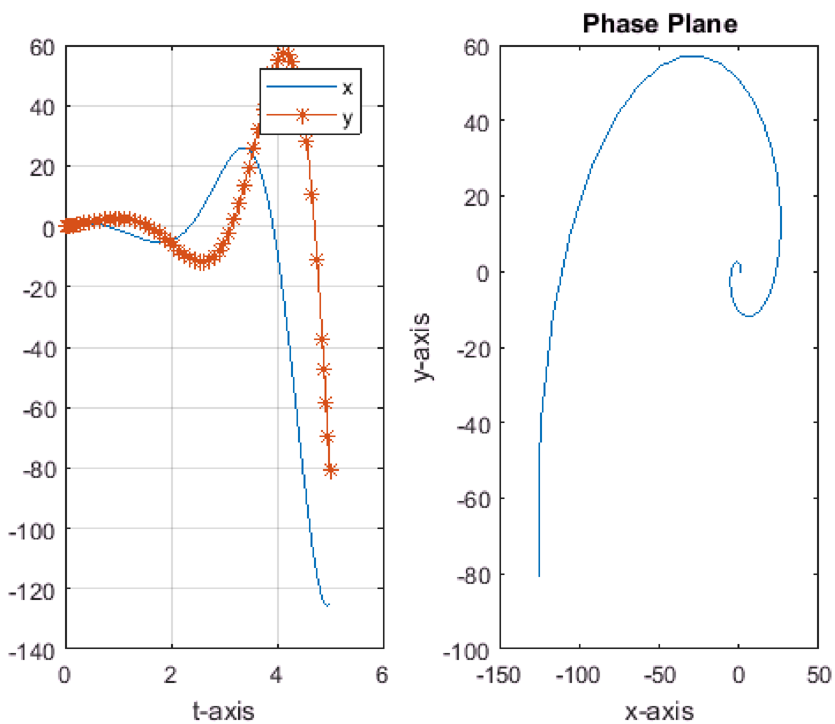
17. $\vec{u} = C_1 e^{t} \begin{bmatrix} \cos(t) \\ \sin(t) \end{bmatrix} + C_2 e^{t} \begin{bmatrix} \sin(t) \\ -\cos(t) \end{bmatrix} + te^{t} \begin{bmatrix} \cos(t) \\ \sin(t) \end{bmatrix}$.

MatLab notes are on the next page.

Math 220 notes for HW#5

I will solve x'=x-2y; y'=y+2x

```
F=@(t,x) [x(1)-2*x(2);x(2)+2*x(1)];
tspan=[0 5];
x0=[1 0];
[t,xa]=ode45(F,tspan,x0);
subplot(1,2,1);
plot(t,xa(:,1))
hold on
plot(t,xa(:,2),'*-')
legend('x', 'y')
xlabel('t-axis')
grid on
hold off
subplot(1,2,2);
plot(xa(:,1),xa(:,2))
xlabel('x-axis')
ylabel('y-axis')
title('Phase Plane')
```



1