20 points Math 220 Homework 7 Sections 4.2 - 4.5, 4.7 18 problems

Most of these problems are from Jiri Lebl's "Notes on Diffy Q's, Differential Equations for Engineers."

1. $f(t) = \sin(t)$ for $t \in (-\pi, \pi]$. Extend $f(t)$ periodically and compute the Fourier series.
2. $f(t) = \sin(\pi t)$ for $t \in (-\pi, \pi]$. Extend $f(t)$ periodically and compute the Fourier series.
3. $f(t) = \sin^2(t)$ for $t \in (-\pi, \pi]$. Extend $f(t)$ periodically and compute the Fourier series.
4. $f(t) = t + t^2$ for $t \in (-2, 2]$. Extend $f(t)$ periodically and compute the Fourier series.
5. $f(t) = \cos(2t)$ for $t \in [0, \pi]$.
   a) Find the Fourier series of the even extension.
   b) Find the fourier series of the odd extension.
6. $f(t) = \sum_{n=1}^{\infty} \frac{1}{n^2} \sin(nt)$. Solve $x'' - x = f(t)$ if $x(0) = 0$ and $x(\pi) = 0$.
7. Solve $u_t = ku_{xx}$ if $u(0,t) = 0$, $u(L,t) = 0$, and $u(x,0) = \begin{cases} 1 & 0 < x < L/2 \\ 0, L/2 < x < L \end{cases}$.
8. Solve $u_t = u_{xx}$ if $u(0,t) = 0$, $u(1,t) = 0$, and $u(x,0) = \sin(\pi x)$ for $0 \le x \le 1$.
9. Use superposition and the answer from the last problem to solve
   $u_t = u_{xx}$ if $u(0,t) = 0$, $u(1,t) = 100$, and $u(x,0) = \sin(\pi x) + 100x$ for $0 \le x \le 1$.
10. Solve $y_{tt} = 25y_{xx}$ if $y(0,t) = y(2,t) = 0$, $y(x,0) = 0$, and $y_t(x,0) = \sin(\pi x) + 0.1\sin(2\pi x)$ for $0 < x < 2$.
11. Solve $y_{tt} = y_{xx}$ if $y_x(0,t) = y_x(1,t) = 0$, $y(x,0) = 0$, and $y_t(x,0) = 2 + \cos(3\pi x) - 4\cos(5\pi x)$ for $0 < x < 1$.
12. Solve $y_{tt} = 9y_{xx}$ if $y_x(0,t) = y_x(\pi,t) = 0$, $y(x,0) = x$, and $y_t(x,0) = 0$ for $0 < x < \pi$.
13. Solve $y_{tt} = 2y_{xx}$ if $y(0,t) = y(\pi,t) = 0$, $y(x,0) = x$, and $y_t(x,0) = 0$ for $0 < x < \pi$.
14. Solve $y_{tt} = y_{xx}$ if $y(0,t) = y(\pi,t) = 0$, $y(x,0) = \sin(x)$, and $y_t(x,0) = 0$ for $0 < x < \pi$.
15. Solve $y_{tt} = y_{xx}$ if $y(0,t) = y(\pi,t) = 0$, $y(x,0) = 0$, and $y_t(x,0) = \sin(x)$ for $0 < x < \pi$.
16. Use superposition and the answers from the last two problems to solve
    $y_{tt} = y_{xx}$ if $y(0,t) = y(\pi,t) = 0$, $y(x,0) = \sin(x)$, and $y_t(x,0) = \sin(x)$ for $0 < x < \pi$.
17. Find a solution to $y_{tt} = 0$ if $y_x(0,t) = y_x(\pi,t) = 0$, $y(x,0) = \sin(2x)$, and $y_t(x,0) = \sin(x)$. In this case, $a = 0$ in $y_{tt} = a^2 y_{xx}$.
18. CAS problem (3 points): use a CAS device for the following. Turn in a pdf file as specified in the syllabus. **Matlab notes for this lab follow the brief answers.**

    Find the first 39 terms of the Fourier series for the 2π-periodic function $g(t) = 1 - \frac{t}{\pi} H(t)$ if $t \in (-\pi, \pi]$, but do not print them! On a single set of axes, plot $g(t)$ with line width 2, and then in normal width the function equal to the sum of the first three terms, the function equal to the sum of the first 21 terms, and the function equal to the sum of the first 39 terms of the Fourier Series. Label them using a legend.

Brief answers

1. $\sin(t)$

2. $\displaystyle\sum_{n=1}^{\infty} \frac{(\pi - n)\sin(\pi n + \pi^2) + (\pi + n)\sin(\pi n - \pi^2)}{\pi n^2 - \pi^3} \sin(nt) = \sum_{n=1}^{\infty} (-1)^n \frac{2n\sin(\pi^2)}{\pi^3 - \pi n^2} \sin(nt)$

3. $\displaystyle\frac{1}{2} - \frac{1}{2}\cos(2t)$

4. $\displaystyle\frac{4}{3} + \sum_{n=1}^{\infty} (-1)^n \left[\frac{16}{n^2\pi^2} \cos\left(\frac{n\pi t}{2}\right) - \frac{4}{n\pi} \sin\left(\frac{n\pi t}{2}\right)\right]$

5. a) $\cos(2t)$ b) $\displaystyle\sum_{n \text{ odd}}^{\infty} \frac{4n}{\pi(n^2 - 4)} \sin(nt) = \sum_{n=1}^{\infty} \frac{4(2n-1)}{\pi\left[(2n-1)^2 - 4\right]} \sin((2n-1)t)$

6. $\displaystyle\sum_{n=1}^{\infty} \frac{-1}{n^2(1 + n^2)} \sin(nt)$

7. $\displaystyle u(x,t) = \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1 - \cos\left(\frac{n\pi}{2}\right)}{n} e^{-kn^2\pi^2 t/L^2} \sin\left(\frac{n\pi x}{L}\right)$

8. $u(x,t) = e^{-\pi^2 t} \sin(\pi x)$

9. $u(x,t) = \sin(\pi x) e^{-\pi^2 t} + 100x$

10. $\displaystyle y(x,t) = \frac{\sin(\pi x)\sin(5\pi t)}{5\pi} + \frac{\sin(2\pi x)\sin(10\pi t)}{100\pi}$

11. $\displaystyle y(x,t) = 2t + \frac{\cos(3\pi x)\sin(3\pi t)}{3\pi} - \frac{4\cos(5\pi x)\sin(5\pi t)}{5\pi}$

12. $\displaystyle y(x,t) = \frac{\pi}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n - 1}{n^2} \cos(nx)\cos(3nt)$

13. $\displaystyle y(x,t) = \sum_{n=1}^{\infty} \frac{2(-1)^n}{n} \sin(nx)\cos(\sqrt{2}\,nt)$

14. $y(x,t) = \sin(x)\cos(t)$

15. $y(x,t) = \sin(x)\sin(t)$

16. $y(x,t) = \sin(x)\left(\sin(t) + \cos(t)\right)$

17. $y(x,t) = \sin(2x) + t\sin(x)$

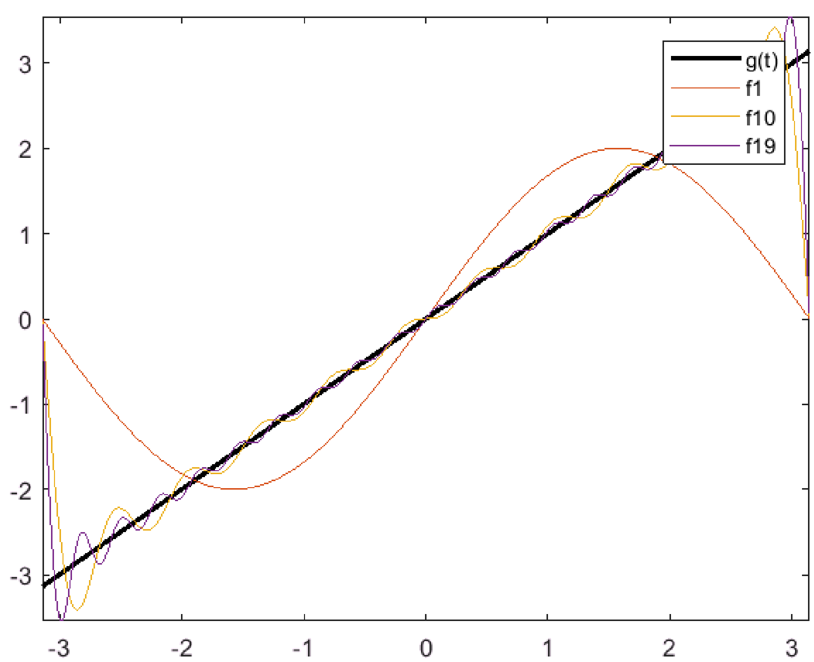
MatLab notes are on the next page.

Notes for HW #7

```
syms g(t)
g(t)=t;
a0=1/pi*int(g(t),t,-pi,pi);
for n=1:20
a(n)=1/pi*int(g(t)*cos(n*t),t,-pi,pi);
b(n)=1/pi*int(g(t)*sin(n*t),t,-pi,pi);
end
f0=a0/2;
y=f0;
for i=1:20
f(i)=y+a(i)*cos(i*t)+b(i)*sin(i*t);
y=f(i);
end
fplot(g(t),[-3.14 3.14],'-k','linewidth',2)
```

```
Warning: MATLAB has disabled some advanced graphics rendering features by switching to software OpenGL. For
 more information, click here.
```

```
hold on
for j=1:9:19
    fplot(f(j),[-3.14 3.14])
end
legend('g(t)','f1','f10','f19')
```



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