20 points Math 220 Homework 8 Sections 4.9, 6.1, 6.2 18 problems

Some problems are from "Notes on Diffy Q's, Differential Equations for Engineers" by Jiri Lebl.

1. Solve $u_{xx} + u_{yy} = 0$ if $u(x, 0) = \sin(9\pi x)$, $u(x, 1) = 0$, $u(0, y) = 0$, and $u(1, y) = 0$ for $0 < x < 1$ and $0 < y < 1$.

2. Solve $u_{xx} + u_{yy} = 0$ if $u(x, 0) = 0$, $u(x, 1) = \sin(2\pi x)$, $u(0, y) = 0$, and $u(1, y) = 0$ for $0 < x < 1$ and $0 < y < 1$.

3. Solve $u_{xx} + u_{yy} = 0$ if $u(x, 0) = \sin(9\pi x)$, $u(x, 1) = \sin(2\pi x)$, $u(0, y) = 0$, and $u(1, y) = 0$ for $0 < x < 1$ and $0 < y < 1$ using superposition and the answers from the last two problems.

4. Solve $u_{xx} + u_{yy} = 0$ if $u_y(x, 0) = 0$, $u_y(x, 1) = 0$, $u(0, y) = 2 + \cos(5\pi y)$, and $u(1, y) = 0$ for $0 < x < 1$ and $0 < y < 1$.

5. Solve $u_{xx} + u_{yy} = 0$ if $u(x, 0) = 0$, $u(x, 1) = 0$, $u_x(0, y) = 0$, and $u_x(1, y) = 2\sin(8\pi y)$ for $0 < x < 1$ and $0 < y < 1$.

6. Solve $u_{xx} + u_{yy} = 0$ if $u_y(x, 0) = 0$, $u_y(x, 1) = 0$, $u(0, y) = 0$, and $u(1, y) = 1 - y$ for $0 < x < 1$ and $0 < y < 1$.

7. Solve $u_{xx} + u_{yy} = 0$ if $u(x, 0) = 3\sin(4x)$, $u(x, 2) = 0$, $u(0, y) = 0$, and $u(\pi/2, y) = 0$ for $0 < x < \pi/2$ and $0 < y < 2$.

8. Solve $u_{xx} + u_{yy} = 0$ if $u(x, 0) = 0$, $u(x, \pi) = 0$, $u(\pi, y) = 1$, and $u_x(0, y) = u(0, y)$ for $0 < x < \pi$ and $0 < y < \pi$.

You may use the table of Laplace Transforms at the end of this homework unless asked to use the definition instead. I use $H(t)$ for the heaviside function, not $u(t)$ that is used in Lebl's book. The heaviside function is also known as the unit-step function.

Use Laplace Transforms to solve the following equations.

9. $q''(t) + 6q'(t) + 8q(t) = 70e^{3t}$ if $q(0) = 1$ and $q'(0) = 2$.

10. $x''(t) - x'(t) = e^t \cos(t)$ if $x(0) = 0 = x'(0)$.

Find the Laplace Transforms of the following functions.

11. $f(t) = t^2 + 6t - 3$

12. $g(t) = e^t \sinh(t)$

Find the Inverse Laplace Transforms of the following functions.

13. $F(s) = \dfrac{4s}{4s^2 + 1}$

14. $G(s) = \dfrac{s}{s^2 + 2s - 3}$

15. Write $f(t) = \begin{cases} 0 & \text{if } t < 1 \\ t-1 & \text{if } 1 \le t < 2 \\ 1 & \text{if } t \ge 2 \end{cases}$ as a linear combination of translates of H(t) and then find its transform.

Use the definition of the Laplace Transform to find $\mathscr{L}(f(t))$ for the given function.

16. $f(t) = \begin{cases} -1 & \text{if } 0 \le t < 1 \\ 1 & \text{if } t \ge 1 \end{cases}$

17. $f(t) = \sinh(at)$

18. CAS problem (3 points): use a CAS device for the following. Turn in a pdf file as specified in the syllabus. **Matlab notes for this lab follow the Laplace Transform table.**

$$y(x,t) = 2t + \frac{\cos(3\pi x)\sin(3\pi t)}{3\pi} - \frac{4\cos(5\pi x)\sin(5\pi t)}{5\pi}$$

is the solution to

$$y_{tt} = y_{xx} \text{ if } y_x(0,t) = y_x(1,t) = 0,\ y(x,0) = 0, \text{ and } y_t(x,0) = 2 + \cos(3\pi x) - 4\cos(5\pi x)$$

for $0 < x < 1$. Make an animation of the solution for $0 \le t \le 2$. Because I won't be able to see an animation on your pdf copy, I want you to turn in a subplot with 16 figures each showing the position of the string at a particular time. These times must be $1/8 = 0.125$ seconds apart. Be sure to give a window with dimensions that do not vary from figure to figure. Your final pdf copy must be only one side of one sheet of paper.

Brief Answers

1. $u(x,y) = \dfrac{\sin(9\pi x)\sinh(9\pi(1-y))}{\sinh(9\pi)}$

2. $u(x,y) = \dfrac{\sin(2\pi x)\sinh(2\pi y)}{\sinh(2\pi)}$

3. $u(x,y) = \dfrac{\sin(2\pi x)\sinh(2\pi y)}{\sinh(2\pi)} + \dfrac{\sin(9\pi x)\sinh(9\pi(1-y))}{\sinh(9\pi)}$

4. $u(x,y) = 2(1-x) + \dfrac{\cos(5\pi y)\sinh(5\pi(1-x))}{\sinh(5\pi)}$

5. $u(x,y) = \dfrac{\sin(8\pi y)\cosh(8\pi x)}{4\pi\sinh(8\pi)}$

6. $u(x,y) = \dfrac{x}{2} + \displaystyle\sum_{n=1}^{\infty} \frac{2(1-(-1)^n)\cos(n\pi y)\sinh(n\pi x)}{n^2\pi^2\sinh(n\pi)}$

7. $u(x,y) = \dfrac{3\sin(4x)\sinh(4(2-y))}{\sinh(8)}$

8. $u(x,y) = \displaystyle\sum_{n=1}^{\infty} \frac{2(1-(-1)^n)\sin(ny)[n\cosh(nx)+\sinh(nx)]}{n\pi[n\cosh(n\pi)+\sinh(n\pi)]}$

9. $q(t) = 3e^{-4t} - 4e^{-2t} + 2e^{3t}$

10. $x(t) = 0.5(1 - e^t \cos(t) + e^t \sin(t)$

11. $\dfrac{2}{s^3} + \dfrac{6}{s^2} - \dfrac{3}{s}$

12. $\dfrac{1}{(s-1)^2 - 1}$

13. $\cos\left(\dfrac{t}{2}\right)$

14. $g(t) = \dfrac{3e^{-3t} + e^t}{4} = e^{-t}[\cosh(2t) - 0.5\sinh(2t)]$

15. $f(t) = (t-1)H(t-1) - (t-2)H(t-2)$ and $F(s) = \dfrac{e^{-s} - e^{-2s}}{s^2}$

16. Prove $F(s) = \dfrac{2e^{-s} - 1}{s}$

17. Prove $F(s) = \dfrac{a}{s^2 - a^2}$

$f(t)$	$F(s)$		$f(t)$	$F(s)$
$f'(t)$	$sF(s) - f(0)$		$\int_0^t f(\tau)g(t-\tau)\,d\tau$	$F(s)G(s)$
$f''(t)$	$s^2F(s) - sf(0) - f'(0)$		$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{at}; $e^{at}f(t)$	$\dfrac{1}{s-a}$; $F(s-a)$		$\int_0^t f(\tau)\,d\tau$	$\dfrac{F(s)}{s}$
$f(t-a)H(t-a)$	$e^{-as}F(s)$		$\cos(at)$	$\dfrac{s}{s^2+a^2}$
t^n	$\dfrac{n!}{s^{n+1}}$		$\sin(at)$	$\dfrac{a}{s^2+a^2}$
$f(t)\delta(t-a)$	$f(a)e^{-as}$		$\cosh(at)$; $\sinh(at)$	$\dfrac{s}{s^2-a^2}$; $\dfrac{a}{s^2-a^2}$

MatLab notes are on the next page.

Matlab notes for HW#8

Here is an animation of the motion a string with Dirichlet boundary conditions - that means the ends are fixed. The frames in this animation are 1/8 second apart so that I can get 16 plots over two minutes. I use the subplot command to plot them all efficiently.

```
syms y(x)
for i=0:0.125:1.875
    subplot(4,4,8*i+1)
   y(x)=5/(2*pi)*sin(2*pi*x)*sin(6*pi*i)- ...
    3/(8*pi)*sin(8*pi*x)*sin(24*pi*i);
    fplot(y,[0 1])
    axis([0 1 -2 2])
    title(['t = ',num2str(i)])
    drawnow
end
```