20 points Math 220 Homework 10 Chapter 7, Notes 18 problems

1. Solve $x^2y'' - 2y = 0$
2. Solve $x^2y'' + xy' + 4y = 0$
3. Solve $25x^2y'' + 25xy' + y = 0$
4. Solve $x^2y'' - xy' + y = 2x$
5. Solve $x^2y'' + 3xy' = 0$ if $y(1) = 0$ and $y'(1) = 4$
6. Solve $xy'' + y' = x$ if $y(1) = 1$ and $y'(1) = -0.5$
7. Find two power series solutions about the ordinary point $(0, 0)$ for $y'' - 2xy' + y = 0$.
8. Find two power series solutions about the ordinary point $(0, 0)$ for $y'' + x^2y' + xy = 0$.
9. Find the singular points. Classify them as regular or irregular.

   a) $x^3y'' + 4x^2y' + 3y = 0$ b) $(x^2 - 9)^2y'' + (x + 3)y' + 2y = 0$

10. Use the Frobenius method to find the general solution of $2xy'' - y' + 2y = 0$ for $x > 0$.
11. Use the Frobenius method to find the general solution of $3xy'' + (2 - x)y' - y = 0$ for $x > 0$.
12. Use the Frobenius method to find the general solution of $xy'' + 2y' - xy = 0$ for $x > 0$. Notice that the indicial roots differ by an integer, but we can still find two linearly independent series solutions about $x = 0$.

Find the general solution of the following equations for $x > 0$.

13. $x^2y'' + xy' + (x^2 - 1/9)y = 0$
14. $4x^2y'' + 4xy' + (4x^2 - 25)y = 0$
15. $xy'' + y' + xy = 0$
16. $x^2y'' + xy' + (9x^2 - 4)y = 0$
17. $x^2y'' + xy' + (25x^2 - 4/9)y = 0$
18. CAS problem (3 points): use a CAS device for the following. Turn in a pdf file as specified in the syllabus. **No Matlab notes are needed for this problem. You should be able to do it using prior notes on prior homework assignments.**

    A) Find the general solution to $xy'' + y' + xy = 0$. Then find the particular solution for the initial values $y(1) = 1$ and $y'(1) = 1$. Notice the singularity makes us avoid $x = 0$.
    B) Use the code in the Matlab documentation to graph the first five Bessel functions of the first kind.
    C) Use the code in the Matlab documentation to graph the first five Bessel functions of the second kind.

Brief Answers

1. $y = C_1 x^{-1} + C_2 x^2$
2. $y = C_1 \cos(2\ln(x)) + C_2 \sin(2\ln(x))$
3. $y = C_1 \cos(0.2\ln(x)) + C_2 \sin(0.2\ln(x))$
4. $y = C_1 x + C_2 x \ln(x) + x(\ln(x))^2$
5. $y = 2 - 2x^{-2}$
6. $y = 0.75 - \ln(x) + 0.25x^2$
7. $y_1 = 1 - \dfrac{x^2}{2!} - \dfrac{3x^4}{4!} - \dfrac{21x^6}{6!} - \ldots$ and $y_2 = x + \dfrac{x^3}{3!} + \dfrac{5x^5}{5!} + \dfrac{45x^7}{7!} + \ldots$
8. $y_1 = 1 - \dfrac{x^3}{3!} + \dfrac{16x^6}{6!} - \dfrac{49 \cdot 16x^9}{9!} + \ldots$ and $y_2 = x - \dfrac{4x^4}{4!} + \dfrac{100x^7}{7!} - \dfrac{6400x^{10}}{10!} + \ldots$
9. a) 0 is irregular b) 3 is irregular and -3 is regular
10. $y = C_1 x^{1.5}\left(1 - \dfrac{2x}{5} + \dfrac{4x^2}{70} - \dfrac{8x^3}{1890} + \ldots\right) + C_2\left(1 + 2x - 2x^2 + \dfrac{4x^3}{9} - \ldots\right)$
11. $y = C_1 x^{1/3}\left(1 + \dfrac{x}{3} + \dfrac{x^2}{18} + \dfrac{x^3}{162} + \ldots\right) + C_2\left(1 + \dfrac{x}{2} + \dfrac{x^2}{10} + \dfrac{x^3}{80} + \ldots\right)$
12. $y = C_1 x^{-1} \displaystyle\sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!} + C_2 x^{-1} \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!} = x^{-1}\left(C_1 \sinh(x) + C_2 \cosh(x)\right)$
13. $y = C_1 J_{1/3}(x) + C_2 Y_{1/3}(x)$
14. $y = C_1 J_{5/2}(x) + C_2 Y_{5/2}(x)$
15. $y = C_1 J_0(x) + C_2 Y_0(x)$
16. $y = C_1 J_2(3x) + C_2 Y_2(3x)$
17. $y = C_1 J_{2/3}(5x) + C_2 Y_{2/3}(5x)$