

Angular Frequency

If the damping coefficient $\alpha = 0$, then we have **free motion**

$$mx'' + kx = 0$$

Solve the equation, and write the final answer as a single cosine function with a phase shift.

$\omega_0 = \sqrt{\frac{k}{m}}$ is the **natural (angular) frequency** with units $\frac{\text{cycles}}{2\pi \text{sec}}$.

Frequency, f , is also described using $\text{Hz} = \frac{\text{cycles}}{\text{sec}}$. Then $\omega_0 \frac{\text{cycles}}{2\pi \text{sec}} \implies f = \frac{\omega_0}{2\pi} \text{Hz}$.

Example: $\omega_0 = 5 \implies f = \frac{5}{2\pi} \text{Hz}$

How does frequency change with damping? Let $\frac{\alpha}{m} = 2p$ and show that $\omega = \sqrt{\omega_0^2 - p^2}$ if the oscillations are underdamped.

Find expressions for the natural frequency and the frequency for the circuit equation written in terms of current¹

$$Li'' + Ri' + \frac{1}{C}i = 0$$

¹Differentiate both sides of $Lq'' + Rq' + \frac{1}{C}q = 0$ to get the current equation remembering that $q' = i$.