Wave Equation

The one-dimensional **wave equation** is

$$\frac{\partial^2 y(x,t)}{\partial t^2} = k^2 \frac{\partial^2 y(x,t)}{\partial x^2}$$

or

$$y_{tt}(x,t) = k^2\, y_{xx}(x,t)$$

where k is a positive constant and $y(x,t)$ is the vertical displacement of a wave at position x and time t. We assume there is no horizontal displacement, and that the vertical displacements are small. Good examples are the string on a violin or an atom vibrating in a molecule.

The Dirichlet boundary conditions model fixed ends to the string; Neumann boundary conditions model free ends. This equation has **two** initial conditions, one for the initial position and the other for initial velocity.

Solve $y_{tt} = k^2 y_{xx}$ if B.C.: $y(0,t) = 0 = y(L,t)$ and I.C.: $y(x,0) = f(x)$, $y_t(x,0) = 0$.

For the rest of the problems, you decide what the initial condition should be. For instance, in the next problem, what is $g(x)$?

Solve $y_{tt} = k^2 y_{xx}$ if B.C.: $y(0,t) = 0 = y(L,t)$ and I.C.: $y(x,0) = 0$, $y_t(x,0) = g(x)$.

Solve $y_{tt} = k^2 y_{xx}$ if B.C.: $y(0,t) = 0 = y(L,t)$ and I.C.: $y(x,0) = f(x)$, $y_t(x,0) = g(x)$. Use superposition.

Solve $y_{tt} = k^2 y_{xx}$ if B.C.: $y_x(0,t) = 0 = y_x(L,t)$ and I.C.: $y(x,0) = 0$, $y_t(x,0) = g(x)$.

Solve $y_{tt} = k^2 y_{xx}$ if B.C.: $y_x(0,t) = 0 = y_x(L,t)$ and I.C.: $y(x,0) = f(x)$, $y_t(x,0) = 0$.