Laplace's 2D Static Heat Equation

The 3-D Heat Equation is

$$u_t = k(u_{xx} + u_{yy} + u_{zz}) = k\Delta u.$$

If the heat distribution is static, $u_t(x, y, z, t) = 0$ so that the equation becomes $\Delta u = 0$. This is Laplace's 3-D static heat equation.

In 2-D,

$$u_{xx}(x, y) + u_{yy}(x, y) = 0.$$

This equation models a rectangular frying pan's static heat distribution. Amazingly, we can find the heat distribution in the interior of the pan if we only know the temperature on the four boundaries.

Solve $u_{xx} + u_{yy} = 0$ if $u(0, y) = 0 = u(a, y)$, $u(x, 0) = 0$, and $u(x, b) = f(x)$.

(cont.)

$u_{xx} + u_{yy} = 0$ if $u(x, 0) = 0 = u(x, b)$, $u(0, y) = 0$, and $u(a, y) = f(y)$ is solved in the same way, so if we intechange the parameters appropriately, we should be able to write down the solution.

For the rest of this lecture, you choose the initial condition. In the next problem, what shall we make $f(y)$?

Solve $u_{xx} + u_{yy} = 0$ if $u(x, 0) = 0 = u(x, b)$, $u(0, y) = f(y)$, and $u(a, y) = 0$.

Write the general soultion for $u_{xx} + u_{yy} = 0$ if $u(0, y) = 0 = u(a, y)$, $u(x, 0) = f(x)$, and $u(x, b) = 0$ without much work using the answer to the last example.

Solve $u_{xx} + u_{yy} = 0$ if $u_y(x, 0) = 0 = u_y(x, b)$, $u(0, y) = 0$, and $u(a, y) = f(y)$

Solve $u_{xx} + u_{yy} = 0$ if $u_x(0, y) = 0 = u_x(a, y)$, $u(x, 0) = f(x)$, and $u(x, b) = 0$

How can we solve $u_{xx} + u_{yy} = 0$ if $u_x(0, y) = f(y)$, $u_x(a, y) = g(y)$, $u(x, 0) = h(x)$, and $u(x, b) = k(x)$?