

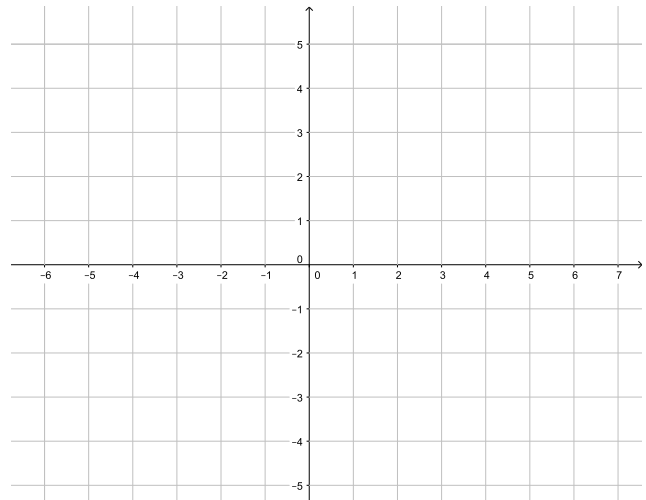
1. (10 points) Find the general solution for  $y'' + y = \sec(x)$ .

2. (10 points) Find the explicit solution for  $x^2y' = y^2 + xy$ ,  $y(1) = 1$ , using the substitution  $v = \frac{y}{x}$ . Show work.

3. (10 points) A tank initially contains 100 liters of water with no salt. A solution containing 0.25 kg of salt per liter is pumped into the tank at the rate of 4 liters per second, and the mixture - kept uniform by stirring - is pumped out at 3 liters per second. **Set up a first order differential equation** in terms of the amount of salt at any time  $t$  and **solve it**.
4. (9 points) Find the charge on the capacitor in the LRC circuit with  $L = 0.1$  Henrys,  $R = 1.5$  Ohms, and  $C = 0.2$  Farads if  $q(0) = 0$  Coulombs,  $q'(0) = 0$  Amperes, and the electromotive force is  $E(t) = 2\delta(t-1)$  Volts. Set up and solve a differential equation using Laplace transforms.

5. (8 points) Find and classify all of the critical points for  $\begin{cases} x' = y(2 - x) \\ y' = x(1 + x - y) \end{cases}$ .

6. (4 points)  $(2, 3)$  is one of the critical points for  $\begin{cases} x' = y(2 - x) \\ y' = x(1 + x - y) \end{cases}$ . Sketch the phase diagram for this system near  $(2, 3)$  using the linearization provided by the Jacobian matrix from the last problem.



7. (9 points) Use the Frobenius method to find the first three nonzero terms of **one** series solution for  $xy'' - xy' - 5y = 0$ .

8. (8 points) Find the general solution for  $\begin{cases} x' = -3x + y + 6e^{-2t} \\ y' = 4y + 12e^{-2t} \end{cases}$ .

9. (6 points) Solve  $x^2y'' + 3xy' + y = 0$ , if  $y(1) = 1$  and  $y'(1) = 1$ .

10. (10 points) Solve  $y_{tt} = 4y_{xx}$  if  $y(0, t) = 0$ ,  $y(1, t) = 0$ ,  $y(x, 0) = 0$ , and  $y_t(x, 0) = x$ . Show all steps.

11. (8 points) Solve  $u_t = 4u_{xx}$  if  $u_x(0, t) = 0$ ,  $u_x(\pi, t) = 0$ , and  $u(x, 0) = \cos^2(x)$ . Show all steps.

12. (8 points) Find the eigenvalues and eigenfunctions for  $\frac{d^2 x}{dt^2} + \lambda x = 0$ , if  $x(a) = 0$ ,  $x'(b) = 0$ , and  $0 < a < b$ . Show all steps.