

1) Find the general solution for  $y'' + 4y' + 13y = 4\sin(x)$ . Show work. (10 points)

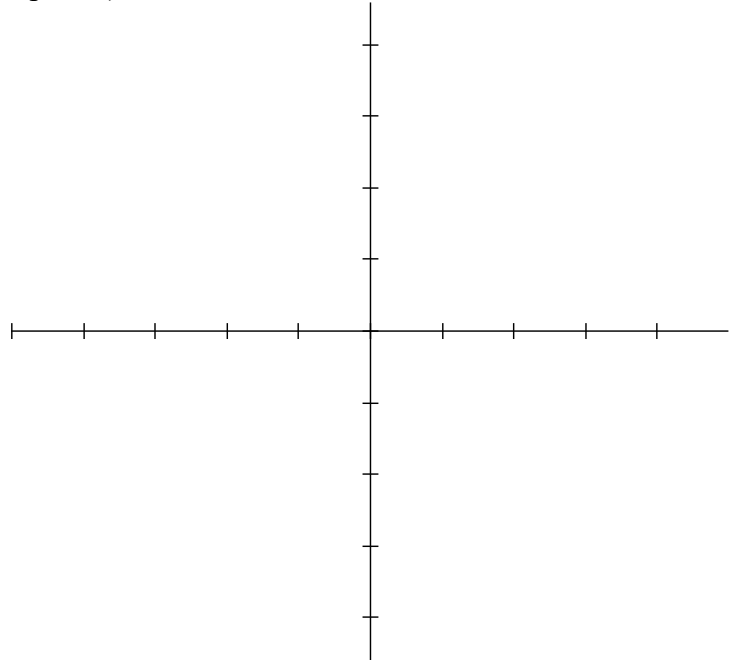
2) Solve  $x^2 y'' + 5xy' - 5y = 0$ ,  $y(1) = 3$ , and  $y'(1) = -3$ . Show work. (5 points)

3) Write the general solution for  $x^2 y'' + xy' + (4x^2 - 25)y = 0$ . No work is required. (3 points)

4) A tank initially contains 100 liters of water with no salt. A solution containing 0.1 kg of salt per liter is pumped into the tank at the rate of 4 liters/sec, and the mixture - kept uniform by stirring - is pumped out at 3 liters/sec. **Set up a first order differential equation** in terms of the amount of salt at any time  $t$  **and solve it.** (10 points)

5) Find the charge on the capacitor in the LRC circuit with  $L = 0.1$  henrys,  $R = 2.5$  ohms,  $C = 0.1$  farads,  $q(0) = 0$  coulombs, and  $i(0) = 0$  amperes, and if the electromotive force is  $E(t) = 3\delta(t-1)$ . Set up and solve a differential equation using Laplace transforms. (10 points)

6) A 2-periodic function  $f(x)$  is defined by  $f(x) = 2H(x+1/2) - 2H(x-1/2)$  when  $-1 < x < 1$ .  $H(x)$  is the unit step function. **Find all the coefficients for the Fourier series of  $f(x)$ , write the sum of the first four nonzero terms, and sketch its graph for  $-2 \leq x \leq 2$ .** (10 points)



7) Find and classify all of the equilibrium points for  $\frac{dx}{dt} = y$  . Show work. (8 points)  
 $\frac{dy}{dt} = -y - \sin x$

8) Use the Frobenius method to find the first three nonzero terms of **one** series solution for  $xy'' + xy' - 9y = 0$ . (8 points)

9) Use the eigenvalue method to find the general solution of  $\begin{cases} \frac{dx}{dt} = 2x - y \\ \frac{dy}{dt} = x + 4y \end{cases}$ . Show work. (8 points)

10) The general solution to a wave equation  $y_{tt} = 25y_{xx}$  is  $y(x, t) = \sum_{n=1}^{\infty} c_n \sin(n\pi x / 3) \sin(5n\pi t / 3)$ .

10A) Find the solution if  $y_t(x, 0) = 5\pi \sin(2\pi x) + 10\pi \sin(5\pi x)$ . (6 points)

10B) What boundary and other initial conditions besides  $y_t(x, 0) = 5\pi \sin(2\pi x) + 10\pi \sin(5\pi x)$  produced the general solution? (2 points)

11) Use the steps discussed in class to solve  $u_t = 4u_{xx}$  with  $u(x, 0) = 3 + 8\cos(5x)$ ,  $u_x(0, t) = 0 = u_x(2\pi, t)$ . (6 points)

12) Use the steps from class to solve  $u_{xx} + u_{yy} = 0$  with  $u(0, y) = 0 = u(1, y)$ ,  $u(x, \pi) = 1$ , and  $u_y(x, 0) = 0$ . (7 points)

13) Find a **particular solution** for  $\vec{u}' = A\vec{u} + \begin{bmatrix} e^x \\ 1 \end{bmatrix}$  if  $\vec{u}_c = A\vec{u}_c$  has solution  $\vec{u}_c = \begin{bmatrix} e^x & 1 \\ -e^x & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$ . (7 points)

*a prime should be here, not here*