Quiz 2 25 points Math 220 Name: ______________________

1. (5 points) Find the general explicit real solution for $(D^2 - 1)(D^2 + D - 2)y = 0$ where D is the differential operator.

$(r^2 - 1)(r^2 + r - 2) = 0$

$(r+1)(r-1)(r+2)(r-1) = 0$

$\boxed{\Rightarrow y = c_1 e^{-x} + c_2 e^{x} + c_3 x e^{x} + c_4 e^{-2x}}$

2. (5 points) Solve $y'' - 6y' + 10y = 0$, $y(0) = 2$ and $y'(0) = -1$.

$r^2 - 6r + 10 = 0$

$r = \frac{6 \pm \sqrt{36 - 40}}{2}$

$r = 3 \pm i$

$y = e^{3x}(c_1 \cos(x) + c_2 \sin(x))$.

$y(0) = 2 \Rightarrow c_1 = 2$

$y'(0) = -1 \Rightarrow 3 \cdot 2 + c_2 = -1$

$\Rightarrow c_2 = -7$.

$\boxed{\therefore y = e^{3x}(2\cos(x) - 7\sin(x))}$

3. (5 points) Find the value of m that makes $\overbrace{my\cos(xy)}^{M}dx + (\overbrace{2y + x\cos(xy)}^{N})dy = 0$ into an exact equation and then find a general **implicit** solution.

The equation is exact if $\frac{\partial(my\cos(xy))}{\partial y} = \frac{\partial(2y + x\cos(xy))}{\partial x}$

$\Rightarrow m[\cos(xy) - xy\sin(xy)] = \cos(xy) - xy\sin(xy) \Rightarrow \boxed{m = 1}$

$\phi = \int y\cos(xy)\,\partial x = \sin(xy) + f(y)$.

Then $x\cos(xy) + f'(y) = \phi_y = N = 2y + x\cos(xy) \Rightarrow f(y) = y^2 + c$.

$\therefore$ the solution is $\boxed{\sin(xy) + y^2 = c}$

Q2

4. (6 points) Use a substitution to solve the IVP $y' - yx = y^2 x$, $y(1) = 1$, **explicitly**.

$$\frac{y'}{y^2} - \frac{x}{y} = x \; ; \quad w = y^{-1} \Rightarrow w' = -y^{-2}y'$$

$$\Rightarrow -w' + xw = x \; ; \quad u = e^{\int x\,dx} = e^{\frac{x^2}{2}}.$$

$$\Rightarrow \frac{d\left[we^{\frac{x^2}{2}}\right]}{dx} = -xe^{\frac{x^2}{2}}$$

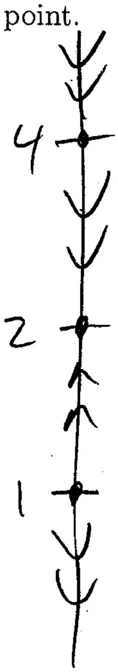
$$\Rightarrow we^{\frac{x^2}{2}} = -e^{\frac{x^2}{2}} + c$$

$$\Rightarrow y^{-1} = -1 + ce^{-\frac{x^2}{2}}.$$

$$y(1) = 1 \Rightarrow 1 = -1 + ce^{-\frac{1}{2}} \Rightarrow c = 2e^{\frac{1}{2}}.$$

$$\boxed{\therefore\; y = \frac{1}{2e^{\frac{1-x^2}{2}} - 1}}$$

5. (4 points) Draw the phase line diagram for $y' = (y-4)^2(y-2)(1-y)$ and determine the stability of each critical point.



C.p.'s

$y = 4$ is unstable (or semistable)

$y = 2$ is stable

$y = 1$ is unstable