

1. (6 points) Solve $x^2 \frac{d^2 y}{dx^2} + 7x \frac{dy}{dx} + 25y = 0$ if $y(1) = 0$ and $y'(1) = 8$.

2. Without doing any work, write the general solution for the following equations.

(a) (2 points) $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + (9x^2 - 25)y = 0$.

(b) (2 points) $x \frac{d^2 y}{dx^2} + \frac{dy}{dx} + \left(x - \frac{16}{x}\right)y = 0$.

3. (3 points) Find the singular points for $(x - 2)^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + (x - 2)y = 0$, and then classify them as regular or irregular.

4. (12 points) Find one solution to $x\frac{d^2y}{dx^2} - \frac{dy}{dx} + xy = 0$. Use the first three nonzero terms of a series solutions.