Quiz 8 20 points Math 220 Name: ______________________

1. (8 points) Solve $u_{xx} + u_{yy} = 0$ if $0 = u_y(x, 0) = u_y(x, \pi) = u(1, y)$ and $u(0, y) = y^2$. Show the steps discussed in lecture.

2. (6 points) Use the improper integral definition of the Laplace transform to find $\mathcal{L}(f(t))$ if $f(t) = \begin{cases} t & 0 \leq t \leq 1 \\ 1 & t \geq 1 \end{cases}$. Do **not** use the table of Laplace transforms in your work.

3. (6 points) Use Laplace transforms to solve $y''(t) + 3y'(t) + 2y(t) = 2H(t-4)$ if $y(0) = 0$ and $y'(0) = 1$. You may use the table of Laplace transforms that I have attached on the next page.

Laplace Transform Table

You may use this table of Laplace Transforms unless asked to use the definition instead. I use $H(t)$ for the heaviside function, not $u(t)$ that is used in Lebl's book. The heaviside function is also known as the unit-step function.

$f(t)$	$F(s)$		$f(t)$	$F(s)$
$f'(t)$	$sF(s) - f(0)$		$\int_0^t f(\tau)g(t-\tau)\,d\tau$	$F(s)G(s)$
$f''(t)$	$s^2F(s) - sf(0) - f'(0)$		$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{at}; $e^{at}f(t)$	$\frac{1}{s-a}$; $F(s-a)$		$\int_0^t f(\tau)\,d\tau$	$\frac{F(s)}{s}$
$f(t-a)H(t-a)$	$e^{-as}F(s)$		$\cos(at)$	$\frac{s}{s^2+a^2}$
t^n	$\frac{n!}{s^{n+1}}$		$\sin(at)$	$\frac{a}{s^2+a^2}$
$f(t)\delta(t-a)$	$f(a)e^{-as}$		$\cosh(at)$; $\sinh(at)$	$\frac{s}{s^2-a^2}$; $\frac{a}{s^2-a^2}$