Quiz 9 10 points Math 220 Name: ____________________

1. (4 points) Solve $y'' + 6y' + 9y = \cos(\pi t)\delta(t-1)$ if $y(0) = 0 = y'(0)$.

$$\overset{\mathcal{L}}{\Longrightarrow} \quad s^2 Y + 6sY + 9Y = \cos(\pi \cdot 1)e^{-s}$$

$$\Rightarrow \quad Y(s+3)^2 = -1e^{-s}$$

$$\Rightarrow \quad Y = \frac{-e^{-s}}{(s+3)^2} \;;$$

This is a t-axis translation signaled by e^{-s} and an s-axis translation of $\frac{1}{s^2}$.

$$\Rightarrow \quad y(t) = -f(t-1)H(t-1) \quad \text{where} \quad \mathcal{L}(f(t)) = \frac{1}{(s+3)^2}.$$

$$f(t) = e^{-3t}\mathcal{L}^{-1}\left(\frac{1}{s^2}\right) = e^{-3t} \cdot t$$

$$\boxed{\therefore \quad y(t) = -e^{-3(t-1)}(t-1)H(t-1)}$$

2. (4 points) Solve $y(t) = t^3 + \int_0^t \sin(t-\tau)y(\tau)\,d\tau$ for $y(t)$.

$$\overset{\mathcal{L}}{\Longrightarrow} \quad Y(s) = \frac{3!}{s^4} + \frac{1}{s^2+1} \cdot Y(s)$$

$$\Rightarrow \quad Y(s)\underbrace{\left[1 - \frac{1}{s^2+1}\right]}_{= \frac{s^2}{s^2+1}} = \frac{3!}{s^4}$$

$$\Rightarrow \quad Y(s) = \frac{3!}{s^6}(s^2+1) = \frac{3!}{s^4} + \frac{3!}{s^6} \cdot \frac{4 \cdot 5}{4 \cdot 5}$$

$$\overset{\mathcal{L}^{-1}}{\Longrightarrow} \quad \boxed{y(t) = t^3 + \frac{t^5}{20}}$$

3. (2 points) Write $f(t) = \begin{cases} 1 & t < 1 \\ 1+t & 1 \le t < 4 \\ 0 & t \ge 4 \end{cases}$ as a linear combination of translates of $H(t)$ and then find its Laplace transform.

$$f(t) = 1 + \underset{\substack{f(t-1) \\ \Rightarrow f(t) = t+1}}{t}\, H(t-1) - \underset{\substack{f(t-4) \\ \Rightarrow f(t) = 5+t}}{(1+t)}\, H(t-4)$$

$$\Rightarrow \quad \boxed{F(s) = \frac{1}{s} + \left(\frac{1}{s^2} + \frac{1}{s}\right)e^{-s} - \left(\frac{5}{s} + \frac{1}{s^2}\right)e^{-4s}}$$

✓ Good!