Quiz 9 25 points Math 220 Name: ______________________

1. (7 points) Solve $y_{tt} = 4y_{xx}$ if $y_x(0,t) = 0$, $y_x(3,t) = 0$, $y(x,0) = 0$, and $y_t(x,0) = 2 + \cos(5\pi x)$. Show ~~all~~ steps. discussed in class

① $y = XT \Rightarrow T'' + 4\lambda T = 0,\ T(0) = 0\ ;\ X'' + \lambda X = 0,\ X'(0) = 0 = X'(3).$

② $\lambda = \frac{n^2\pi^2}{9},\ n = 0,1,2,\cdots;\ X_n = \cos\left(\frac{n\pi}{3}x\right),\ T_0 = t,\ T_n = \sin\left(\frac{2n\pi t}{3}\right)$

③ $y(x,t) = c_0 t + \sum_{n=1}^{\infty} c_n \cos\left(\frac{n\pi x}{3}\right)\sin\left(\frac{2n\pi t}{3}\right)$

④ $2 + \cos(5\pi x) = y_t(x,0) = c_0 + \sum_{n=1}^{\infty} \frac{2n\pi}{3} c_n \cos\left(\frac{n\pi x}{3}\right)$

$\Rightarrow c_0 = 2,\ c_{15} = \frac{3}{30\pi}$, and $c_n = 0$ otherwise.

$$\boxed{\therefore\ y(x,t) = 2t + \frac{1}{10\pi}\cos(5\pi x)\sin(10\pi t)}$$

2. (4 points) Use past work to help solve $y_{tt} = 4y_{xx}$ if $y_x(0,t) = 0$, $y_x(3,t) = 0$, $y(x,0) = \cos(\pi x)$, and $y_t(x,0) = 0$.

$T'' + 4\lambda T = 0;\ T'(0) = 0 \Rightarrow T_n = \cos\left(\frac{2n\pi}{3}t\right)$

$\Rightarrow y(x,t) = c_0 + \sum_{n=1}^{\infty} c_n \cos\left(\frac{n\pi x}{3}\right)\cos\left(\frac{2n\pi t}{3}\right)$

$\cos(\pi x) = y(x,0) = c_0 + \sum_{n=1}^{\infty} c_n \cos\left(\frac{n\pi x}{3}\right) \Rightarrow c_3 = 1,\ c_n = 0$ for $n \neq 3$.

$$\therefore\ \boxed{y(x,t) = \cos(\pi x)\cos(2\pi t)}$$

3. (2 points) Use past work to write a solution for $y_{tt} = 4y_{xx}$ if $y_x(0,t) = 0$, $y_x(3,t) = 0$, $y(x,0) = \cos(\pi x)$, and $y_t(x,0) = 2 + \cos(5\pi x)$.

$$\boxed{y(x,t) = 2t + \cos(\pi x)\cos(2\pi t) + \frac{1}{10\pi}\cos(5\pi x)\sin(10\pi t)}$$

Q9

4. (8 points) Solve $u_{xx} + u_{yy} = 0$ if $u(0,y) = 4 + \cos(3y)$, $u(1,y) = 0$, $u_y(x,0) = 0$, and $u_y(x,\pi) = 0$. Show ~~all~~ steps discussed in class.

① $u = XY \Rightarrow Y'' + \lambda Y = 0,\ Y'(0) = 0 = Y'(\pi)\ ;\ X'' - \lambda X = 0,\ X(1) = 0.$ (translated ½ Dirichlet)

② $\lambda = n^2,\ n = 0,1,2,\dots;\ Y_n = \cos(ny);\ X_0 = (1-x),\ X_n = \sinh(n(1-x))$

③ $u(x,y) = C_0(1-x) + \sum_{n=1}^{\infty} C_n \cos(ny)\sinh(n(1-x))$

④ $4 + \cos(3y) = u(0,y) = C_0 + \sum_{n=1}^{\infty} C_n \sinh(n)\cos(ny)$

$\Rightarrow C_0 = 4,\ C_3 = \frac{1}{\sinh(3)}.$

$$\boxed{\therefore\ u(x,y) = 4(1-x) + \frac{\cos(3y)\sinh(3(1-x))}{\sinh(3)}}$$

5. (4 points) Use prior work to help solve $u_{xx} + u_{yy} = 0$ if $u(0,y) = y$, $u(1,y) = 0$, $u_y(x,0) = 0$, and $u_y(x,\pi) = 0$.

From #4, $u(x,y) = C_0(1-x) + \sum_{n=1}^{\infty} C_n \cos(ny)\sinh(n(1-x))$

$y = u(0,y) = C_0 + \sum_{n=1}^{\infty}(C_n\sinh(n))\cos(ny) \Rightarrow C_0 = \frac{a_0}{2},\ C_n = \frac{a_n}{\sinh(n)}$

F.S. u_E: $b_n = 0$; $a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} u_E\cos(ny)\,dy = \frac{2}{\pi}\int_0^{\pi} y\cos(ny)\,dy$

+	y	$\cos(ny)$
−	1	$\frac{\sin(ny)}{n}$
+	0	$-\frac{\cos(ny)}{n^2}$

$= \frac{2}{\pi}\left[\frac{\cos(ny)}{n^2}\right]_0^{\pi} = \frac{2}{n^2\pi}\left((-1)^n - 1\right).$

$a_0 = \frac{2}{\pi}\int_0^{\pi} y\,dy = \frac{y^2}{\pi}\Big|_0^{\pi} = \pi.$

$$\boxed{\therefore\ u(x,y) = \frac{\pi}{2}(1-x) + \sum_{n=1}^{\infty}\frac{2\left[(-1)^n - 1\right]}{n^2\pi\sinh(n)}\cos(ny)\sinh(n(1-x))}$$