

1. (10 points) Solve $u_{xx} + u_{yy} = 0$ if $u(x, 0) = 0$, $u(x, 1) = 0$, $u(0, y) = \sin(2\pi y)$, and $u(3, y) = 0$. Show the steps discussed in class.
2. (6 points) Solve $u_{xx} + u_{yy} = 0$ if $u(x, 0) = 0$, $u(x, 1) = 0$, $u(0, y) = 0$, and $u(3, y) = 4 \sin(5\pi y)$. You may skip steps.
3. (2 points) Write the solution for $u_{xx} + u_{yy} = 0$ if $u(x, 0) = 0$, $u(x, 1) = 0$, $u(0, y) = \sin(2\pi y)$, and $u(3, y) = 4 \sin(5\pi y)$. You do not need to show any work.

4. (9 points) Solve $y''(t) + 6y'(t) + 8y = (t^2 - 1)\delta(t - 2)$ if $y(0) = 2$ and $y'(0) = 1$.

5. (8 points) Solve $y(t) = H(t - 2) - \int_0^t (t - \tau)y(\tau) d\tau$.

6. (9 points) Find the eigenvalues and eigenfunctions for $x''(t) + \lambda x(t) = 0$, $x(0) = 0$, and $x'(\pi) = 0$. Show detailed work.

7. (8 points) Solve $u_t = u_{xx}$ if $u(0, t) = 0$, $u(2, t) = 0$, and $u_t(x, 0) = x$. Show the steps discussed in class.

8. (10 points) Solve $y_{tt} = 4y_{xx}$ if $y_x(0, t) = 0$, $y_x(1, t) = 0$, $y(x, 0) = 1 - 2 \cos(\pi x)$, and $y_t(x, 0) = 0$. Show the steps discussed in class.

9. (6 points) Use past work to help solve $y_{tt} = 4y_{xx}$ if $y_x(0, t) = 0$, $y_x(1, t) = 0$, $y(x, 0) = 0$, and $y_t(x, 0) = 1 - 2 \cos(\pi x)$.

10. (8 points) Use the definition of Laplace transform to show that $\mathcal{L}\{f'(t)\} = s\mathcal{L}\{f(t)\} - f(0)$.

11. (8 points) Find $\mathcal{L}^{-1}\left\{\frac{s-1}{s^2(s^2+1)}\right\}$.

12. (9 points) Let $f(x) = H(x - \pi)$ for $0 \leq x \leq 2\pi$. Find the first three nonzero terms of the cosine series expansion for $f(x)$.

13. (9 points) Solve $u_t = u_{xx}$ if $u_x(1, t) = 0$, $u_x(2, t) = 0$, and $u(x, 0) = \cos(\pi x)$. Show the steps discussed in class.