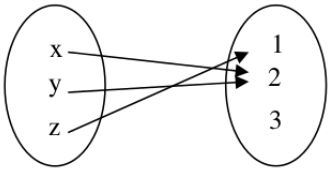
Math 137
Exam #2 Review Guide

The second exam will cover Sections 2.1-2.8. The problems on this review guide are representative of the type of problems worked on homework and during class time. Do not just depend on this guide for studying for the exam. When you have trouble with a particular problem type, you should go back to the text, homework, and class notes to find additional problems to practice. For the problem types you are comfortable with, you should still practice some more, in addition to this guide. The answers to the following problems are attached. ***Make sure you are in the habit of showing all your work; you will need to do so on the exam to receive credit.***

Chapter 2

1. Let $A = \{x, y, z\}$ and $B = \{1, 2, 3\}$. A rule of association, f, is shown in the figure.



   a. Does the rule represent a function?

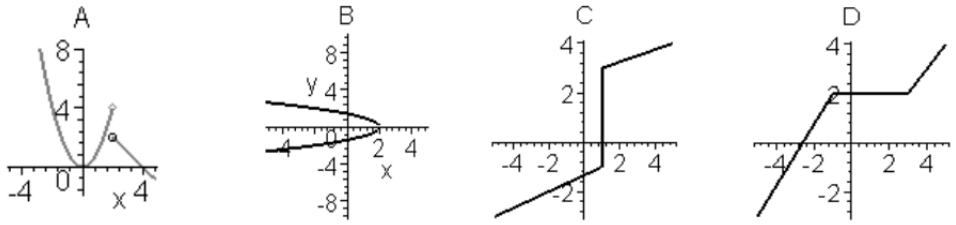
   b. If the rule does represent a function, determine the range of f.

2. Determine the domain of the function $y = \dfrac{1}{x^2 - 4}$.

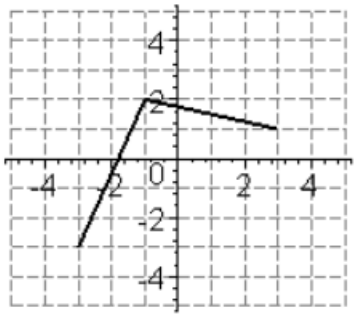
3. Determine the domain and range of the function $y = \sqrt{2x + 1}$.

4. Determine the domain of the function $f(x) = \sqrt{\dfrac{x+3}{x-5}}$.

5. Which of the following graphs represent functions?



6. Determine the domain and range of the function shown below.



7. Let $f(x) = \begin{cases} x^2 & x < 0 \\ 2x + 1 & x \geq 0 \end{cases}$. Sketch the graph of $f(x)$.

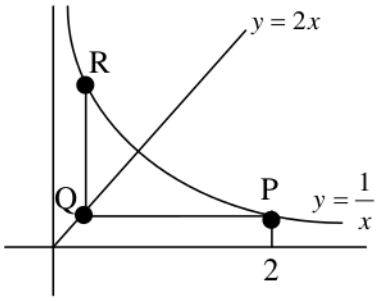
8. Let $h(x) = \begin{cases} -x+2 & x < -1 \\ x^2 - 1 & -1 \le x < 1 \\ \sqrt{x+1} & 1 \le x \end{cases}$.

a. Compute $h(0)$.

b. Compute $h(1)$.

c. Compute $h(2) - h(-2)$.

9. Determine the coordinates of P, Q, and R in the figure shown below.



10. Let $A = \{2, 4, 7, 8, 11, 23\}$ and let f be the rule that assigns to each number in A the closest multiple of 3. Is this rule a function? If so, give the range.

11. Let $f(x) = 2x^2 + 1$. Evaluate each of the following.

a. $f(-1)$

b. $f(3)$

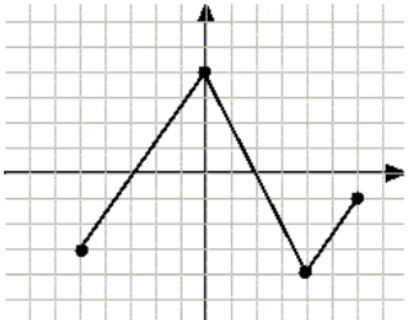
c. $f(2+h)$

12. Determine the domain of $f(x) = \dfrac{1}{x^4 - 2x^2 - 3}$. Use interval notation.

13. Use a graph to determine the domain of $f(x) = \sqrt{x^2 - 4}$. Use interval notation.

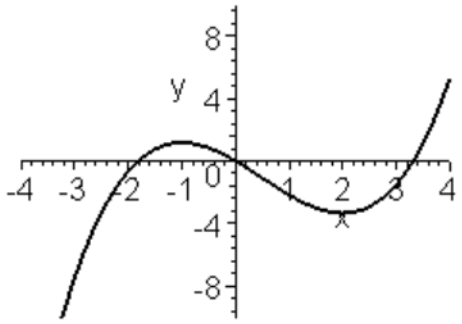
14. Use a graph to find the range of the function $f(x) = \dfrac{2x-1}{x+3}$. Use interval notation.

15. Determine the domain and range of the function shown below.

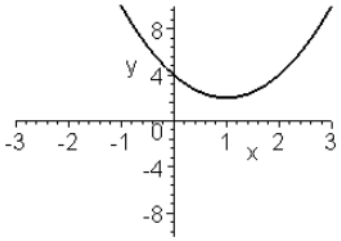


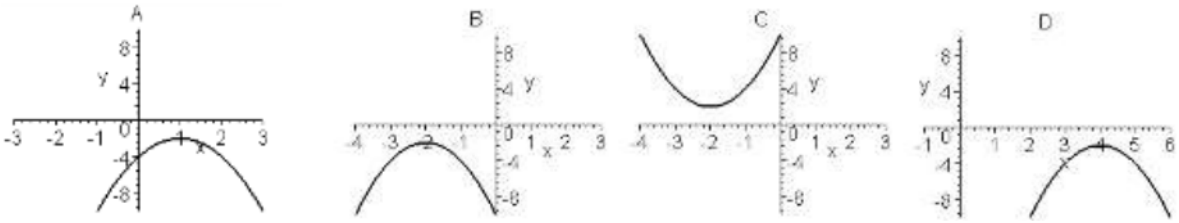
16. Let $f(x) = \dfrac{x+1}{x-2}$. Evaluate and simplify $f(a+3)$.

17. A portion of the graph of a function f defined on the entire real line is shown below. Determine the interval(s) on which f is increasing. Assume nothing unusual occurs outside the interval from -4 to 4.



18. The graph of a function f is shown to the right. Which of the following is the graph of $-f(x+3)$?





19. The graph of a function g is the line segment joining the points $(-2, 0)$ and $(3, 1)$. The graph of $g(x+1)+2$ is also a line segment joining two points A and B. Find the points A and B.

20. Let $f(x) = x^2 + 1$ and $g(x) = 3x - 2$.

    a. Compute $\left(\dfrac{f}{f+g}\right)(2)$.

    b. Compute $\left(\dfrac{f-g}{fg}\right)(1)$.

21. Let $f(x) = x + \dfrac{1}{x}$ and $g(x) = 2x + 1$.

    a. Find $(f \circ g)(x)$

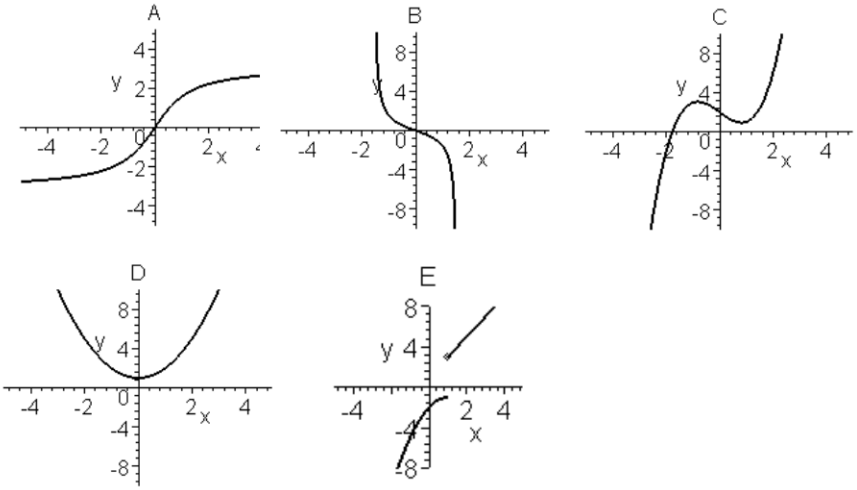
    b. Find $(g \circ f)(x)$

22. Let $f(x) = 2x^2 - 1$. Compute $(f \circ f)(-2)$

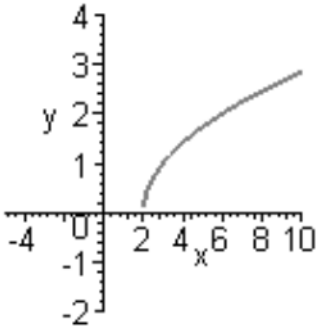
23. If the function $f(x) = \dfrac{x^2 + x^4}{1 + x^2}$ is written in the form $(h \circ g)(x)$ where $g(x) = x^2$, find $h(x)$.

24. Find $f^{-1}(x)$ if $f(x) = \dfrac{2x + 1}{3x - 1}$.

25. Which of the following graphs represent one-to-one functions?



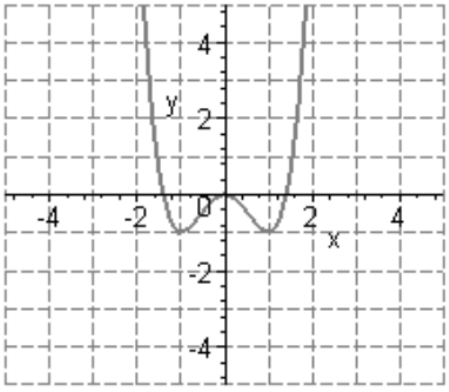
26. The graph of a function g is shown below. Sketch the graph of $y = -g(-x)$.



27. The graph of a function g is obtained from the graph of another function f by translating the graph of f to the right 3 units and then translating the resulting graph 2 units downward. Give a formula for $g(x)$ in terms of f.

28. Determine whether the function $y = \dfrac{2x}{1 + 3x^2}$ is even, odd, or neither.

29. The graph of $y = x^4 - 2x^2$ is shown below. Determine the intervals on which the function is decreasing.



30. Determine the maximum value of the function $y = 3x^2 - 1$ with domain $-3 \leq x \leq 2$.

31. Let $f(x) = 2x^2 + x + 1$. Compute and simplify $\dfrac{f(x+h) - f(x)}{h}$.

32. Let $y = \begin{cases} \sqrt{x} & 0 \leq x \leq 1 \\ \frac{1}{x} & 1 < x \leq 2 \end{cases}$. Sketch the graph of this function and determine its maximum value.

33. Let $f(x) = 3x - 1$ and $g(x) = \dfrac{1}{x+1}$. Compute $(2f - 3g)(2)$.

34. Let $f(x) = x^2 - 1$ and $g(x) = \sqrt{x}$. Give a formula for $(f \circ g)(x)$ and state the domain of this function.

35. Which of the following functions is an even function? There may be more than one.

    a. $y = x^2 + 1$

    b. $y = x^3 - 1$

    c. $y = \dfrac{x}{x^2 + 1}$

    d. $y = x^2 + |x|$

    e. $y = 2x - 3x^2$

36. Which of the following functions is one-to-one? There may be more than one.

    a. $y = x^2$ on $-1 \leq x \leq 1$

    b. $y = x^2$ on $-1 \leq x \leq 2$

    c. $y = x^2$ on $0 \leq x \leq 2$

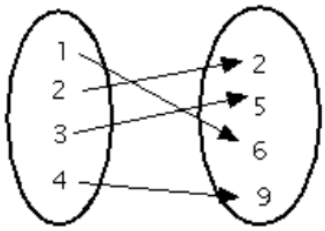
    d. $y = x^2$ on $2 \leq x \leq 3$

    e. $y = x^2$ on $-3 \leq x \leq 1$

37. If $f(x) = \frac{1}{2}x + \frac{2}{3}$, find $f^{-1}\left(\frac{3}{4}\right)$.

38. The graph of a function f is the line segment joining the points $(-1, 4)$ and $(2, -3)$. The graph of f^{-1} is also a line segment joining the two points A and B. What are the points A and B?

39. A function is shown to the right. Evaluate each of the following.



    a. $f^{-1}(2)$

    b. $f^{-1}(9)$

    c. $f(f^{-1}(5))$

40. Let $f(x) = \dfrac{x^3+1}{x^3-1}$. Find a formula for $f^{-1}(x)$. You may assume that f is one-to-one on its domain.

41. Use a graph to determine the domain of the function $y = \sqrt{x^2+x-6}$. Use interval notation.

42. The height (in feet) of an object t seconds after being fired upwards from the ground is given by $h(t) = 80t - 16t^2$. Determine the maximum height attained by the object.

43. Express the volume V of a cube as a function of its total surface area S.

44. A right circular cylinder has its height equal to 4 times the radius of its base. Express the volume V of the cylinder as a function of its radius r.

45. Find a linear function f satisfying $f(1) = 4$ and $f(3) = -2$.

46. A factory owner purchases a new machine for \$32,000. After 10 years, the machine has a salvage value of \$4,000. Determine a formula for the value of the machine t years after being purchased, where $0 \le t \le 10$.

47. Find a linear function $f(x)$ that passes through the point $(1, 4)$ and increases 5 units for every two-unit increase in x.

48. A new car is purchased for \$24,000. After 10 years, the car is worth \$2,000. Assuming the car depreciates at a constant rate, find the value of the car after 6 years.

49. Let $f(x) = x + \dfrac{2}{x}$. Determine the average rate of change of $f(x)$ over the interval $[1, 3]$.

50. Let $f(x) = \dfrac{1}{x+1}$. Find $\dfrac{f(x+h)-f(x)}{h}$ and simplify it to an expression that is defined when $h = 0$.

51. Determine the range of $(f \circ g)(x)$ if $f(x) = \dfrac{1}{x^2}$ and $g(x) = \dfrac{1}{2x-1}$.

52. Compute the average rate of change of the function $y = x^2 + 2x + 2$ over the interval $0 \le x \le 2$.

Chapter 1 Answers

1. (a) yes (b) $\{1, 2\}$

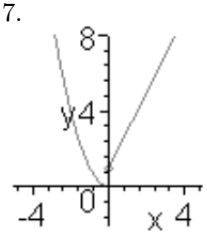
2. $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

3. D: $\left[-\frac{1}{2}, \infty\right)$, R: $[0, \infty)$

4. $(-\infty, -3] \cup (5, \infty)$

5. a and d

6. D: $[-3, 3]$, R: $[3, 2]$

7. 

8. (a) -1 (b) $\sqrt{2}$ (c) $\sqrt{3} - 4$

9. $P\left(2, \frac{1}{2}\right), Q\left(\frac{1}{4}, \frac{1}{2}\right), R\left(\frac{1}{4}, 4\right)$

10. It is a function; R: $\{3, 6, 9, 12, 24\}$

11. (a) 3 (b) 19 (c) $2h^2 + 8h + 9$

12. $(-\infty, -\sqrt{3}) \cup (-\sqrt{3}, \sqrt{3}) \cup (\sqrt{3}, \infty)$

13. $(-\infty, -2] \cup [2, \infty)$

14. $(-\infty, 2) \cup (2, \infty)$

15. D: $[-5, 6]$, R: $[-4, 4]$

16. $\frac{a+4}{a+1}$

17. $(-\infty, -1) \cup (2, \infty)$

18. B

19. $(-3, 2)$ and $(2, 3)$

20. (a) $\frac{5}{9}$ (b) $\frac{1}{2}$

21. (a) $2x + 1 + \frac{1}{2x+1}$ (b) $2x + \frac{2}{x} + 1$

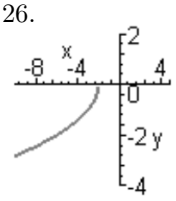
22. 97

23. $\frac{x+x^2}{1+x}$

24. $f^{-1}(x) = \frac{x+1}{3x-2}$

25. a, b, e

26.



27. $g(x) = f(x-3) - 2$

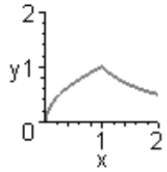
28. odd

29. $(-\infty, -1] \cup [0, 1]$

30. 26

31. $4x + 2h + 1$

32.



33. 9

34. $(f \circ g)(x) = x - 1$, D: $x > 0$

35. a, d

36. c, d

37. $\frac{1}{6}$

38. $(4, -1), (-3, 2)$

39. 2, 4, 5

40. $\sqrt[3]{\frac{x+1}{x-1}}$

41. $(-\infty, -3] \cup [2, \infty)$

42. 100 ft

43. $V(S) = \left(\frac{S}{6}\right)^{3/2}$

44. $V(r) = 4\pi r^3$

45. $f(x) = -3x + 7$

46. $V(t) = 32000 - 2800t$

47. $f(x) = \frac{5}{2}x + \frac{3}{2}$

48. \$10,800

49. $\frac{1}{3}$

50. $\frac{-1}{(x+h+1)(x+1)}$

51. $(0, \infty)$

52. 4