Math 137
Exam #3 Review Guide

The third exam will cover Sections 3.1-3.6, 4.1-4.7. The problems on this review guide are representative of the type of problems worked on homework and during class time. Do not just depend on this guide for studying for the exam. When you have trouble with a particular problem type, you should go back to the text, homework, and class notes to find additional problems to practice. For the problem types you are comfortable with, you should still practice some more, in addition to this guide. The answers to the following problems are attached. ***Make sure you are in the habit of showing all your work; you will need to do so on the exam to receive credit.***

Review of Polynomial & Rational Inequalities

1. Solve each polynomial inequality, writing your solutions using interval notation.

a. $x^2 + 5x - 6 < 0$

b. $10x - x^2 \leq 24$

c. $(x-3)(x+1)(x+5) \geq 0$

d. $x^4(x-2)(x-16) < 0$

2. Solve each rational inequality, writing your solutions using interval notation.

a. $\dfrac{7-x}{3-4x} \geq 0$

b. $\dfrac{x^2-1}{x^2+10x+24} \geq 0$

c. $\dfrac{1}{x-2} - \dfrac{1}{x-1} < \dfrac{1}{6}$

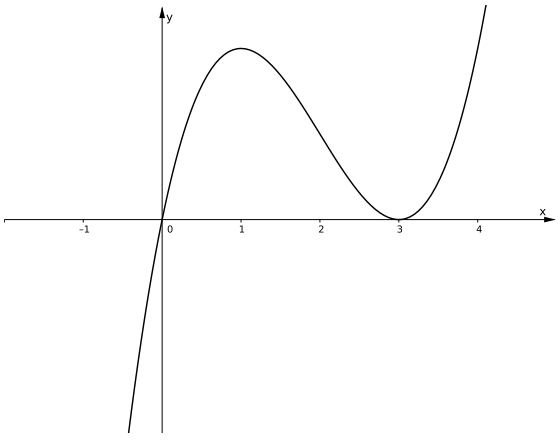
Chapter 3

1. How many times does the graph of the polynomial $y = x^3(x-1)^2(x+4)(x-3)^4$ cross the x-axis? Explain your answer.

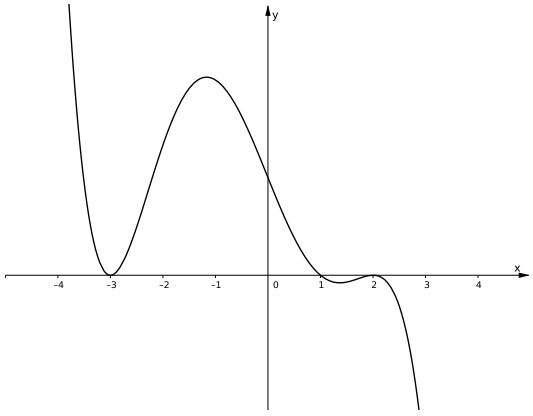
2. Sketch the graph of the polynomial $y = 2x^2 - x^4$. Be sure to label the intercepts and discuss any symmetry of the graph.

3. Give a polynomial of lowest degree that has the given graph.

a.



b.



4. Find the end-behavior, intercepts, and other important features of the following and sketch their graphs.

   a. $P(x) = -x^2(x-3)$

   b. $P(x) = (x+2)(x-1)^2(x-3)^2$

5. Find all horizontal and vertical asymptotes of each function.

   a. $f(x) = \dfrac{2x}{2x^2 - 5x - 3}$

   b. $f(x) = \dfrac{(2x-1)(3x+5)}{x^2+1}$

6. Sketch the graph of the function $f(x) = \dfrac{12}{(x+4)^2(x-3)}$.

7. Find all values of x where the graph of $f(x) = \dfrac{(x-1)(x-2)}{x^2+1}$ intersects its horizontal asymptote.

8. Give a formula for a rational function having the following properties:

   - Horizontal asymptote: $y = 2$
   - Vertical asymptote: $x = -3, x = 1$
   - x-intercepts: $(-1, 0), (5, 0)$

9. Determine the quotient and remainder using long division.

   a. $(8x^3 - 6x^2 - 7x + 3) \div (2x - 3)$

   b. $(3x^4 - 2x^2 + 5x - 13) \div (x^2 - 1)$

10. In what situations is synthetic division a valid approach to division of polynomials?

11. Determine the quotient and remainder using synthetic division.

    a. $(2x^4 + 6x^3 - 4x^2 - 5x + 7) \div (x + 3)$

    b. $(3x^4 - 4x^2 + 5x + 8) \div (x + 1)$

12. Find a polynomial of lowest degree with zeros $-i$, 3, and -4, where -4 is a root of multiplicity 2.

13. Given that $-2 + i$ is a zero of $P(x) = x^3 + 2x^2 - 3x - 10$, find the other zeros.

14. Determine the vertex of the quadratic function $y = 2x^2 + 4x + 7$.

15. Determine the minimum value of the quadratic function $y = 2x^2 + 7x - 4$.

16. Determine the vertex and axis of symmetry of the quadratic function $y = x^2 - 3x + 11$.

17. Determine the largest value of the quadratic function $y = -2x^2 + 4x + 3$.

18. Let $f(x) = (3x - 2)(x - 4)$. Determine the axis of symmetry and the vertex of this function.

19. Find a quadratic function with vertex $(1, 2)$ that passes through the point $(4, 7)$.

20. Three adjacent corrals are to be built with a total of 1800 ft of fencing. Determine the dimensions (width and total length) of the corrals that will enclose the most area.

21. The height (in feet) of an object t seconds after being fired upwards from the ground is given by $h(t) = 80t - 16t^2$. Determine the maximum height attained by the object.

Chapter 4

1. Simplify the expression $\dfrac{3^{1+2\sqrt{2}}}{9^{1+\sqrt{2}}}$.

2. Solve each equation.

   a. $2^{3x-1} = 8$

   b. $2^x = 8\sqrt{2}$

   c. $9^{2x+1} = 3^{3x-2}$

   d. $3^x x^2 - 4(3^x) = 0$

3. Determine the domain of each function.

   a. $y = \dfrac{1}{2^{x-1} - 1}$

   b. $y = \log_4(x + 5) - 2$

   c. $y = \sqrt{\ln(x - 2) - 1}$

   d. $y = \log_3(6 + x - x^2)$

   e. $y = \ln\left(\dfrac{2x - 1}{x - 3}\right)$

4. Find the range of each function.

   a. $y = 2 + 3^{-x}$

   b. $y = (\ln x)^2 + 1$

5. Write the equation $\log_4(16) = 2$ in exponential form.

6. Express $\log_6(8)$ in terms of natural logarithms.

7. Find a formula for the inverse function $f^{-1}(x)$.

   a. $f(x) = 4e^{2x} + 1$

   b. $f(x) = 4\ln(3x - 2)$

8. Rewrite $\log x - 2\log(x + 1) + 3\log(x - 2)$ as a single logarithm.

9. Expand $\ln\left(\dfrac{\sqrt{x^2+2}}{x^3(x-2)^{3/2}}\right)$ so that the expression contains no logarithms of products, quotients, or powers.

10. Solve $3^{2x-4}=5$, writing your answer in terms of base-3.

11. Solve each equation.

a. $e^{x^2-1}=1$

b. $\log_3(2x+5)=2$

c. $e^{2x}-e^x-2=0$

d. $2^{3x+1}=7$

e. $\ln(x^2-3)=2$

f. $\log_6 x+\log_6(x+1)=1$

g. $20e^{-2.5t}+72=90$

h. $[\ln(1+x)]^2=4$

i. $e^{2x}-3e^x+2=0$

j. $\ln x-\ln(x-2)=\ln 4$

k. $\log_x(2)=3$

12. How does the graph of $y=-2+e^{-x}$ alter the graph of $y=e^x$?

13. How does the graph of $y=-2\ln(x-4)$ alter the graph of $y=\ln x$?

14. Find values of a and b such that the points $(1,2)$ and $(3,7)$ lie on the graph of $f(x)=a\cdot b^x$.

15. Suppose \$2,500 is invested at 5% compounded quarterly. Determine the value of the investment after three and one-half years.

16. Determine the time in years required for an investment to triple when it is invested at 4.5% per annum compounded continuously.

17. How much must one invest initially at 5% per annum compounded continuously so that after 18 years, the investment will be worth \$100,000?

18. Suppose the half-life of a radioactive substance is 4 hours. If one initially has 20 grams of this substance, how many grams will remain after 24 hours?

19. If a particular population doubles in 11.5 years, how long will it take an initial population of 2000 to reach 5500?

20. A bacteria population is measured as 2000 initially and measured as 2850 four hours later. Calculate the doubling time of the population to the nearest hundredth of an hour.

21. A population is modeled by the function $p(t)=\dfrac{500}{2+3e^{-t}}$, where t is time in years. Determine the initial population and describe what happens to the population as t becomes very large.

22. Suppose \$2,000 is invested at 6% per annum. Determine the value of the investment after one year if the interest is compounded quarterly. What if it is compounded monthly?

23. Suppose \$10,000 is invested at 5% per annum compounded monthly. Determine the number of months required for the investment to be worth \$15,000. Round partial months up to the next month.

24. Determine the interest rate per annum required for an investment with continuous compounding to double in 7.5 years. Express your answer as a percentage accurate to two decimal places.

25. A radioactive isotope has a half-life of 30 minutes. How long will it take for there to remain 15% of the original amount?

26. A bacteria population initially contains 3,000 bacteria. After 2 hours, the population has grown to 4,500. Assuming exponential growth, estimate the size of the population after 6 hours.

27. Radioactive carbon-14 has a half-life of 5,730 years. It is determined that a particular relic contains 5% of the original amount of carbon-14 when it was alive. Estimate the age of the relic.

Review of Polynomial & Rational Inequalities Answers

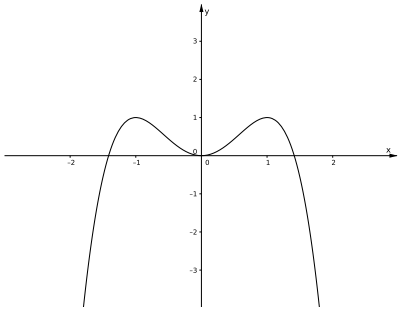
1. a. $(-6, 1)$ b. $(-\infty, 4] \cup [6, \infty)$ c. $[-5, -1] \cup [3, \infty)$ d. $(2, 16)$

2. a. $\left(-\infty, \frac{3}{4}\right) \cup [7, \infty)$ b. $(-\infty, -6) \cup (-4, -1] \cup [1, \infty)$ c. $(-\infty, -1) \cup (1, 2) \cup (4, \infty)$

Chapter 3 Answers

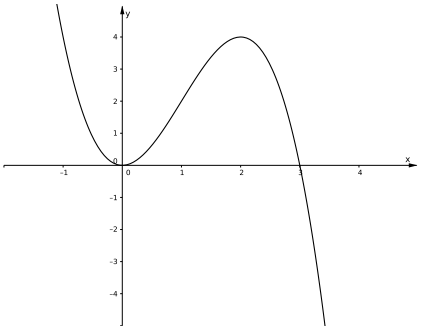
1. 2 times

2. y-int: $(0, 0)$; x-ints: $(0, 0), (\pm\sqrt{2}, 0)$; even function

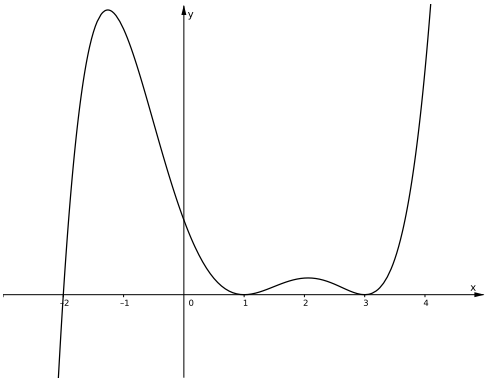


3. a. $y = x(x-3)^2$ b. $y = -(x+3)^2(x-1)(x-2)^2$

4. a.

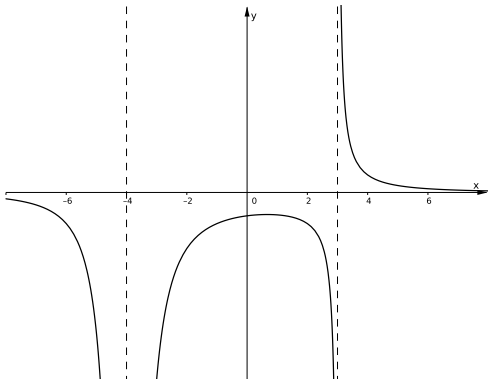


b.



5. a. HA: $y = 0$, VA: $x = -\frac{1}{2}, 3$ b. HA: $y = 6$, VA: none

6.



7. $x = \frac{2}{3}$

8. Possible answer: $y = \frac{2(x+1)(x-5)}{(x+3)(x-1)}$

9. a. $Q = 4x^2 + 3x + 1; R = 6$ b. $Q = 3x^2 + 1; R = 5x - 12$

10. When the divisor is of the form $x - r$

11. a. $Q = 2x^3 - 4x + 7; R = -14$ b. $Q = 3x^3 - 3x^2 - x + 6; R = 2$

12. $P(x) = (x^2 + 1)(x - 3)(x + 4)^2$

13. $-2 - i, 2$

14. $(-1, 5)$

15. $-\frac{81}{8}$

16. Vertex: $\left(\frac{3}{2}, 2\right)$, Axis: $x = \frac{3}{2}$

17. 5

18. Axis: $x = \frac{7}{3}$, Vertex: $\left(\frac{7}{3}, -\frac{25}{3}\right)$

19. $y = \frac{5}{9}(x - 1)^2 + 2$

20. width: 225 ft, length: 450 ft

21. 100 ft

Chapter 4 Answers

1. $\frac{1}{3}$

2. a. $\frac{4}{3}$ b. $\frac{7}{2}$ c. -4 d. ± 2

3. a. $(-\infty, 1) \cup (1, \infty)$ b. $(-5, \infty)$ c. $[2 + e, \infty)$ d. $(-2, 3)$ e. $\left(-\infty, \frac{1}{2}\right) \cup (3, \infty)$

4. a. $(2, \infty)$ b. $[1, \infty)$

5. $4^2 = 16$

6. $\frac{\ln 8}{\ln 6}$

7. a. $f^{-1}(x) = \frac{1}{2} \ln\left(\frac{x-1}{4}\right)$ b. $f^{-1}(x) = \frac{2 + e^{x/4}}{3}$

8. $\log\left(\dfrac{x(x-2)^3}{(x+1)^2}\right)$

9. $\frac{1}{2}\ln(x^2+2) - 3\ln x - \frac{3}{2}\ln(x-2)$

10. $\frac{1}{2}(4+\log_3(5))$

11. a. ± 1 b. 2 c. $\ln(2)$ d. $\frac{1}{3}\left(\dfrac{\ln 7}{\ln 2} - 1\right)$ e. $\pm\sqrt{3+e^2}$ f. 2 g. $\dfrac{-\ln(9/10)}{2.5}$ h. $e^2-1, \dfrac{1}{e^2}-1$ i. $0, \ln 2$ j. $\frac{8}{3}$ k. $\sqrt[3]{2}$

12. reflect about y-axis, shift down 2 units

13. reflect about x-axis, vertical stretch by factor of 2, shift right 4 units

14. $a = \frac{2}{\sqrt{7/2}}, b = \sqrt{7/2}$

15. \$2,974.89

16. 24.41 years

17. \$40,657

18. 0.3125 gram

19. 16.8 years

20. 7.83 hours

21. $p(0) = 100$; $p(t) \to 250$ as $t \to \infty$

22. quarterly: \$2,122.73; monthly: \$2,123.36

23. 98 months

24. 9.24%

25. 82.11 minutes

26. 10,125 bacteria

27. 24,765 years