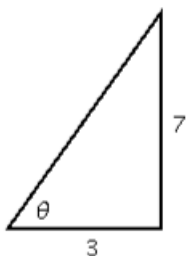


Exam #2 Review Guide (Chapters 6, 7 & 8)

The second exam will cover Sections 6.3, 7.1-7.5 & 8.1-8.2. The problems on this review guide are representative of the type of problems worked on homework and during class time. Do not just depend on this guide for studying for the exam. When you have trouble with a particular problem type, you should go back to the text, homework, and class notes to find additional problems to practice. For the problem types you are comfortable with, you should still practice some more, in addition to this guide. The answers to the following problems are attached. *Make sure you are in the habit of showing all your work; you will need to do so on the exam to receive credit.*

Chapter 6

1. Evaluate $\cos^{-1}\left(-\frac{1}{2}\right)$.
2. Evaluate $\sec\left(\tan^{-1}\sqrt{3}\right)$.
3. Determine the domain of the function $y = \sin^{-1}(2x - 3)$.
4. Evaluate $\sin^{-1}\left(\sin\frac{5\pi}{6}\right)$.
5. Determine the angle θ in degrees where θ is the angle shown in the right triangle below.



6. Determine the range of the function $f(x) = 4 - \cos^{-1}x$.
7. Evaluate $\cos^{-1}\left(\cos\frac{3\pi}{2}\right)$.
8. Simplify the expression $\sin\left(\tan^{-1}\frac{x}{3}\right)$.
9. Find the exact value of $\sin\left(2\cos^{-1}\frac{2}{3}\right)$.
10. Let $f(x) = 3\tan^{-1}\left(\frac{x}{4} + 1\right)$. Find a formula for $f^{-1}(x)$.

Chapter 7

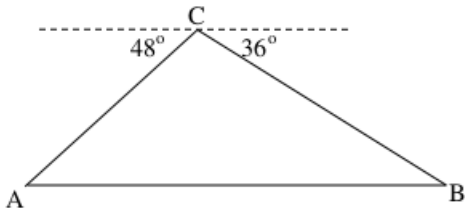
1. Find the exact value of $\sin\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$.
2. Find the exact value of $\cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$.
3. Find the exact value of $\tan\left(\frac{2\pi}{3} + \frac{\pi}{4}\right)$.
4. Given $0 \leq \alpha, \beta \leq \frac{\pi}{2}$, $\sin\alpha = \frac{1}{3}$, and $\sin\beta = \frac{1}{4}$, find the exact value of $\sin(\alpha + \beta)$.
5. Simplify the expression $\sin\left(\theta + \frac{\pi}{4}\right) + \sin\left(\theta - \frac{\pi}{4}\right)$.
6. Given $0 \leq \theta, \phi \leq \frac{\pi}{2}$, $\tan\theta = \frac{1}{2}$, and $\tan\phi = \frac{3}{4}$, find the exact value of $\tan(\phi - \theta)$.
7. Simplify the expression $\frac{\tan\frac{\pi}{4} - \tan\frac{\pi}{12}}{1 + \tan\frac{\pi}{4}\tan\frac{\pi}{12}}$.
8. Given $0 \leq \theta \leq 90^\circ$ and $\sin\theta = \frac{2}{5}$, find the exact value of $\sin 2\theta$.

9. Find the exact value of $\cos 2\theta$ if $\cos \theta = -\frac{1}{3}$.
10. Find the exact value of $\sin 22.5^\circ$.
11. Express $\sin^4 x$ in terms of $\cos 2x$ and $\cos 4x$ with the aid of the double angle formula for the cosine function.
12. Find the exact value of $\tan 15^\circ$.
13. If $x = 3 \sin \theta$ and $0 \leq \theta \leq \frac{\pi}{2}$, express $\sin 2\theta$ in terms of x .
14. Simplify $\cos \frac{7\pi}{24} \cos \frac{3\pi}{72}$.
15. Express $\sin 7x - \sin 3x$ in terms of $\sin 5x$ and $\cos 2x$.
16. Find all solutions of the equation $\sin x = -\frac{\sqrt{3}}{2}$ within the interval $[0, 2\pi]$. Express your answers in radians.
17. Find all solutions of the equation $2 \cos^2 x - 7 \cos x + 3 = 0$ within the interval $[0, 2\pi]$. Express your answers in radians.
18. Find all solutions of the equation $\tan^2 x + 2 \sec^2 x = 7$, where $-\frac{\pi}{2} < x < \frac{\pi}{2}$.
19. Solve the equation $\sin 2\theta = \cos \theta$, where $0 \leq \theta \leq 2\pi$.
20. If $x = 4 \sin \theta$, express $3\theta + 4 \sin 2\theta$ in terms of x .
21. Find all solutions of $\cos x = \frac{1}{3}$ for $0 \leq x \leq 2\pi$.
22. Find the exact value of $\sin \left(\frac{3\pi}{4} - \frac{\pi}{6} \right)$.
23. Find the exact value of $\cos(15^\circ)$.
24. Suppose $\cos x = \frac{1}{3}$ and $\cos y = \frac{2}{5}$, find the exact value of $\cos(x + y)$.
25. Simplify the expression $\sin \left(x - \frac{\pi}{2} \right)$.
26. Find the exact value of $\tan 75^\circ$.
27. Suppose $\tan x = 3$ and $\tan y = 5$. Find the exact value of $\tan(x + y)$.
28. Given $0 \leq \theta \leq \frac{\pi}{2}$ and $\sin \theta = \frac{2}{3}$, find the exact value of $\sin 2\theta$.
29. Given $\frac{\pi}{2} \leq \theta \leq \pi$ and $\sin \theta = \frac{3}{5}$, calculate $\sin \frac{\theta}{2}$.
30. Express the function $\sin^4 2\theta$ in terms of trigonometric functions without powers.
31. Given $0 \leq \theta \leq \frac{\pi}{2}$ and $x = 3 \sin \theta$, express $\cos 2\theta$ in terms of x .
32. Express $\sin 3\theta$ in terms of $\sin \theta$ and $\cos \theta$.
33. Given $\frac{\pi}{2} \leq \theta \leq \pi$ and $\sin \theta = \frac{3}{8}$, find the exact value of $\cos 2\theta$.
34. Express $\sin 3t \sin 5t$ as a sum or difference of cosine functions.
35. Given $0 \leq \theta < \frac{\pi}{2}$ and $x + 3 = \tan \theta$, express $\sin 2\theta$ in terms of x .
36. Find the exact value of $\sin 165^\circ \cos 105^\circ$.

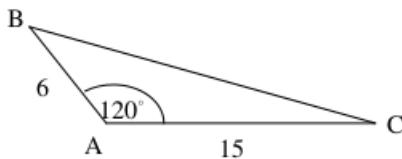
37. Find all solutions of the equation $\sin x = \frac{3}{4}$ for $0 \leq x \leq 2\pi$. Use inverse trigonometric functions to express your answer.
38. Find all solutions of the equation $\sin 2\theta = -\cos \theta$ for $0 \leq \theta \leq 2\pi$.
39. Rewrite the sine wave $\sin 3t + \sqrt{3} \cos 3t$ in the form $A \sin(3t + C)$ for constants A and C .
40. Find all solutions of the equation $\sec \theta = 3.5$ with $0 \leq \theta \leq 2\pi$. Express your answers as decimals accurate to three decimal places.
41. Find all solutions of the equation $\sin x = -\frac{\sqrt{3}}{2}$ within the interval $[0, 2\pi]$. Express your answers in radians.
42. Find all solutions of the equation $2 \cos^2 x - 7 \cos x + 3 = 0$ within the interval $[0, 2\pi]$. Express your answers in radians.
43. Find all solutions of the equation $\tan^2 x + 2 \sec^2 x = 7$, where $-\frac{\pi}{2} < x < \frac{\pi}{2}$.
44. Solve the equation $\sin 2\theta = \cos \theta$, where $0 \leq \theta \leq 2\pi$.
45. Find all solutions of $\cos x = \frac{1}{3}$ for $0 \leq x \leq 2\pi$.
46. Find all solutions of the equation $\sin x = \frac{3}{4}$ for $0 \leq x \leq 2\pi$. Use inverse trigonometric functions to express your answer.
47. Find all solutions of the equation $\sin 2\theta = -\cos \theta$ for $0 \leq \theta \leq 2\pi$.
48. Find all solutions of the equation $\sec \theta = 3.5$ with $0 \leq \theta \leq 2\pi$. Express your answers as decimals accurate to three decimal places.

Chapter 8

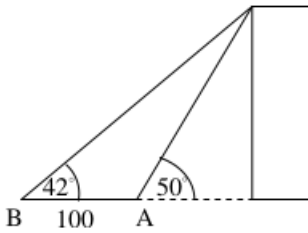
1. A plane flying at a height of 3.5 miles is at point C . At that point, the angle of depression to city A is measured to be 48° and the angle of depression to city B is measured to be 36° . Calculate the distance (in miles) between cities A and B .



2. In $\triangle ABC$, $\angle A = 42^\circ$, $\angle B = 65^\circ$, and $AC = 12$. Find BC .
3. In $\triangle ABC$ shown below, find BC .



4. The angle of elevation to the top of a building at point A is 50° . At point B which is 100 ft farther away from the building than point A , the angle of elevation to the top of the building is 42° . Find the height of the building in feet.



5. Two cars traveling on straight roads leave an intersection at the same time. Car A is traveling at 55 mph while car B is traveling at 65 mph. The angle between the roads is 30° . Determine the distance between the cars 10 minutes after they leave the intersection.

6. A rectangle is inscribed within an octagon of radius 8. Determine the area of the octagon.

7. In $\triangle ABC$, $\angle A = 34^\circ$, $AC = 10$, and $CB = 7$. Show that there are two possible values for $\angle B$. Find both possible values of $\angle B$ in degrees.

8. A rocket is fired from sea level and climbs at a constant angle with the horizontal of 72° . When the rocket is 45,000 feet high, what is the distance from the point at which the rocket was fired and the point directly below the rocket on level ground?

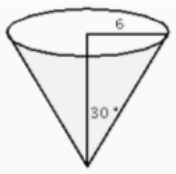
9. In $\triangle ABC$, $\angle A = 30^\circ$, $\angle B = 120^\circ$, and $AB = 10$ ft. Find AC and CB in feet.

10. An isosceles triangle has two equal sides of length 4. The angle between the two equal sides is $\frac{\pi}{4}$ radians. Determine the length of the third side of the triangle.

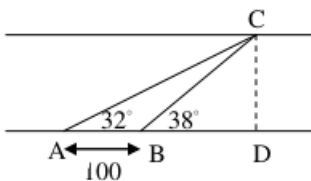
11. In $\triangle ABC$, $AB = 7$, $AC = 5$, and $BC = 6$. Determine $\angle A$. Express your answer both in degrees and in radians.

12. A regular pentagon is inscribed in a circle of radius 7. Determine the area of the pentagon. Give both the exact form of the answer and a decimal approximation that is accurate to two decimal places.

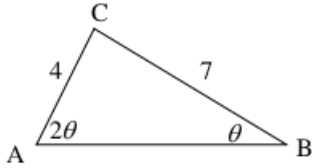
13. A right circular cone has a radius at the top of 6 cm and angle at the vertex of 30° , as shown below. Calculate the volume of the cone. Give the exact value of the volume and a decimal approximation that is accurate to three decimal places. Note that the volume of a cone is $V = \frac{1}{3}\pi r^2 h$.



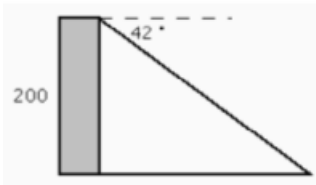
14. A surveyor wants to measure the width of a river. She picks a tree (point C) on the opposite side of the river and determines that $\angle CAB = 32^\circ$. She walks 100 ft along the river to point B and determines that $\angle CBD = 38^\circ$. Determine CD , the width of the river in feet.



15. In the figure below, determine $\angle B$ in degrees.



16. A person standing on a roof of a 200 ft building is looking down at another person on the ground. If the angle of depression is 42° , determine the distance between the bottom of the building and the person on the ground. Express your answer as an exact form and as a decimal accurate to three decimal places.



Chapter 6 Answers

1. $\frac{2\pi}{3}$
2. 2
3. $1 \leq x \leq 2$
4. $-\frac{\pi}{6}$
5. 60.801°
6. $[4 - \pi, 4]$
7. π
8. $\frac{x}{\sqrt{9+x^2}}$
9. $\frac{4\sqrt{5}}{9}$
10. $f^{-1}(x) = 4 \left(\tan \left(\frac{x}{3} \right) - 1 \right)$

Chapter 7 Answers

1. $\frac{\sqrt{6}+\sqrt{2}}{4}$
2. $\frac{\sqrt{6}+\sqrt{2}}{4}$
3. $\frac{1-\sqrt{3}}{1+\sqrt{3}}$
4. $\frac{2\sqrt{2}+\sqrt{15}}{12}$
5. $\sqrt{2} \sin \theta$
6. $\frac{2}{11}$
7. $\frac{\sqrt{3}}{3}$
8. $\frac{4\sqrt{21}}{25}$

9. $-\frac{7}{9}$
10. $\frac{\sqrt{2-\sqrt{2}}}{2}$
11. $\frac{3}{8} - \frac{1}{2} \cos 2x + \frac{1}{8} \cos 4x$
12. $2 - \sqrt{3}$
13. $\frac{2x\sqrt{9-x^2}}{9}$
14. $\frac{1+\sqrt{2}}{4}$
15. $2 \sin 5x \cos 2x$
16. $\frac{4\pi}{3}, \frac{5\pi}{3}$
17. $\frac{\pi}{3}, \frac{5\pi}{3}$
18. $\tan^{-1}\left(\sqrt{\frac{3}{5}}\right), \tan^{-1}\left(-\sqrt{\frac{3}{5}}\right)$
19. $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$
20. $3 \sin^{-1}\left(\frac{x}{4}\right) + \frac{x\sqrt{16-x^2}}{2}$
21. $\cos^{-1}\left(\frac{1}{3}\right), \pi - \cos^{-1}\left(\frac{1}{3}\right),$ or 1.2310, 1.9106
22. $\frac{\sqrt{2}+\sqrt{6}}{2}$
23. $\frac{\sqrt{6}-\sqrt{2}}{2}$
24. $\frac{2-2\sqrt{42}}{4}$
25. $-\cos x$
26. $\frac{\sqrt{3}+1}{\sqrt{3}-1}$ or $2 + \sqrt{3}$
27. $-\frac{4}{7}$
28. $\frac{4\sqrt{5}}{9}$
29. $\frac{3\sqrt{10}}{10}$
30. $\frac{3}{8} - \frac{1}{2} \cos 4\theta + \frac{1}{8} \cos 8\theta$
31. $1 - \frac{2}{9}x^2$
32. $\sin \theta(4 \cos^2 \theta - 1)$
33. $\frac{23}{32}$
34. $\frac{1}{2}(\cos 2x - \cos 8x)$
35. $\frac{2(x+3)}{x^2+6x+10}$
36. $\frac{\sqrt{3}-2}{4}$
37. $\sin^{-1}\left(\frac{3}{4}\right), \pi - \sin^{-1}\left(\frac{3}{4}\right)$

38. $\frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$
39. $2 \sin\left(3t + \frac{\pi}{3}\right)$
40. 1.281, 5.002
41. $\frac{4\pi}{3}, \frac{5\pi}{3}$
42. $\frac{\pi}{3}, \frac{5\pi}{3}$
43. $\tan^{-1}\left(\sqrt{\frac{3}{5}}\right), \tan^{-1}\left(-\sqrt{\frac{3}{5}}\right)$
44. $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$
45. $\cos^{-1}\left(\frac{1}{3}\right), \pi - \cos^{-1}\left(\frac{1}{3}\right)$, or 1.2310, 1.9106
46. $\sin^{-1}\left(\frac{3}{4}\right), \pi - \sin^{-1}\left(\frac{3}{4}\right)$
47. $\frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$
48. 1.281, 5.002

Chapter 8 Answers

1. 7.97 miles
2. 8.86
3. 18.73
4. 368.3 ft
5. 5.42 miles
6. 181.02
7. $53.02^\circ, 126.98^\circ$
8. 14621.39
9. $BC = 10, AC = 10\sqrt{3}$
10. $\frac{2\sqrt{2}}{\sin\left(\frac{3\pi}{8}\right)} \approx 3.0161$
11. 0.997 rad, 57.122°
12. 116.50
13. 391.781
14. 32.04
15. 28.96°
16. 222.1