Exam #3 Review Guide (Chapters 8 & 11)

The third exam will cover Sections 8.3-8.4, 8.6-8.8 & 11.1-11.4. The problems on this review guide are representative of the type of problems worked on homework and during class time. Do not just depend on this guide for studying for the exam. When you have trouble with a particular problem type, you should go back to the text, homework, and class notes to find additional problems to practice. For the problem types you are comfortable with, you should still practice some more, in addition to this guide. The answers to the following problems are attached. ***Make sure you are in the habit of showing all your work; you will need to do so on the exam to receive credit.***

Chapter 8

1. A curve is given parametrically by $x = 2t + 3$ and $y = 3t + 1$. Determine the distance from the origin to the point on the curve when $t = 2$.

2. Sketch the curve given parametrically by the equations $x = 2\cos t$ and $y = -2\sin t$. Indicate the direction of motion.

3. A curve is given parametrically by the equations $x = 1 + 2t$ and $y = 4 + t^2$. Find an equation for the curve in x and y.

4. The parametric equations for a curve in the xy-plane are $x = 2 + t^2$ and $y = 4 - 3t$. Determine the points where the curve intersects the x-axis.

5. Convert the point with polar coordinates $\left(2, \frac{7\pi}{6}\right)$ into rectangular coordinates.

6. Convert the point having rectangular coordinates $(-3, 3)$ into polar coordinates.

7. Find the distance between the points with polar coordinates $\left(2, \frac{\pi}{6}\right)$ and $\left(3, \frac{5\pi}{12}\right)$.

8. Sketch the curve with polar equation $r = 2 + 3\sin\theta$.

9. Convert the equation $y = 4x^2$ into a polar equation. Also give the domain of θ.

10. Convert the polar equation $r^2 = 2\cos^2\theta + 3\sin^2\theta$ into rectangular coordinates.

11. A spiral is given in polar coordinates by the equation $r = Ae^{B\theta}$, $-\infty < \theta < \infty$. If the spiral passes through the points with polar coordinates $(1, 0)$ and $(4, \pi)$, determine A and B.

12. A curve is given parametrically by the equations $x = 2 + t^2$ and $y = 1 + 2t$. Let P be the point on the curve when $t = 1$ and let Q be the point on the curve when $t = 2$. Determine which of the points P or Q is farther from the origin.

13. The parametric equations of a curve in the plane are $x = 4 - t^2$ and $y = 3 + t$. Determine the points in the plane where the curve intersects the y-axis.

14. A curve is given parametrically by the equations $x = -\cos t$ and $y = 2\sin t$, where $0 \leq t \leq 2\pi$. Sketch the curve. Give the starting point and indicate the direction of motion as t increases.

15. A line in the plane is given parametrically by the equations $x = 2 + 3t$ and $y = 3 - 2t$. Find the slope-intercept form of the equation of the line.

16. Give the polar coordinates of the point having rectangular coordinates $(1, \sqrt{3})$.

17. The polar equation of a line is $r\cos\left(\theta - \frac{\pi}{3}\right) = 6$. Determine the rectangular coordinates of the point on the line that is closest to the origin.

18. Sketch the curve with polar equation $r = \frac{2\theta}{\pi}$,where $0 \leq \theta \leq 2\pi$.

19. Explain why the curve with polar equation $r = 2 + \sin\theta + \cos^2\theta$ is not symmetric with respect to the x-axis.

20. Find the rectangular equation for the curve with polar equation $r = \frac{2}{1-\sin\theta}$.

21. Consider the points $P(1,2)$ and $Q(4,-3)$. Determine the magnitude of $\overrightarrow{PQ}$.

22. Two forces $\mathbf{F} = 4\mathbf{N}$ and $\mathbf{G} = 6\mathbf{N}$ act on a particle. Suppose $\mathbf{F}$ acts in the positive x-direction while $\mathbf{G}$ acts in the positive y-direction. Determine the angle θ in degrees between $\mathbf{F} + \mathbf{G}$ and the positive x-direction.

23. If $\mathbf{a} = \langle -3, 1\rangle$ and $\mathbf{b} = \langle 2, 5\rangle$, determine the vector $2\mathbf{a} - 3\mathbf{b}$.

24. If $\mathbf{a} = \langle 3, 5\rangle$, find a vector $\mathbf{u}$ of length 1 and having the same direction as $\mathbf{a}$. Also find a vector $\mathbf{v}$ of length 2 and having the same direction $\mathbf{a}$.

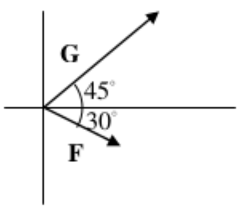
25. If $\mathbf{a} = \langle 1, 2\rangle$ and $\mathbf{b} = \langle -1, 3\rangle$, determine the vector $|\mathbf{b}|(\mathbf{a} + \mathbf{b})$.

26. Express the vector $3\langle 1, 2\rangle - 2\langle -1, 2\rangle$ in tems of the unit vectors $\mathbf{i}$ and $\mathbf{j}$.

27. Consider the points $P(4,3)$ and $Q(-1,5)$. Determine the magnitude of the vector $\overrightarrow{PQ}$.

28. Consider the points $P(1,2)$, $Q(3,5)$, and $R(5,9)$. Which of the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ has the larger angle between the vector and the positive x-direction? Explain how you arrived at your answer.

29. Two forces $\mathbf{F} = 5$ lb and $\mathbf{G} = 10$ lb are applied as shown in the diagram. Find the magnitude of the resulting force $\mathbf{F} + \mathbf{G}$.



30. If $\mathbf{a} = \langle -1, 2\rangle$ and $\mathbf{b} = \langle 2, 3\rangle$, determine the vector $\mathbf{a} - 2\mathbf{b}$.

31. Find a unit vector $\mathbf{u}$ having the same direction as the vector $\mathbf{b} = \langle -5, 3\rangle$.

32. Let $\mathbf{a} = \langle -2, 3\rangle$ and $\mathbf{b} = \langle 4, 7\rangle$. Find a vector $\mathbf{c}$ such that $\mathbf{a} + 2\mathbf{c} = 3\mathbf{b}$.

33. If $\mathbf{a} = \langle -2, 3\rangle$ and $\mathbf{b} = \langle 4, 6\rangle$, determine $|\mathbf{a} + \mathbf{b}|$.

34. Determine the angle (in degrees) between the vectors $\mathbf{a} = \langle 2, 3\rangle$ and $\mathbf{b} = \langle 2, 6\rangle$.

Chapter 11

1. Given the formula $1 + 3 + 5 + \cdots + (2n - 1) = n^2$, evaluate $1 + 3 + 5 + \cdots + 31$.

2. Given the formula $1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$, find $6 + 7 + 8 + \cdots + 20$.

3. Let $f(n) = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^n}$. Then $f(1) = \frac{1}{2}$, $f(2) = \frac{3}{4}$, $f(3) = \frac{7}{8}$, and $f(4) = \frac{15}{16}$. Based on these results, conjecture a fractional formula for $f(n)$.

4. Simplify $\frac{(2n+1)!}{(2n-1)!}$.

5. Suppose the sequence $a_a, a_2, a_3, \ldots$ is defined by $a_n = \frac{n}{n+2}$ for $n \geq 1$. Compute the sum of the first three terms in the sequence. Express your answer as a single fraction.

6. Suppose the sequence $a_a, a_2, a_3, \ldots$ is defined by the recursion equation $a_{n+1} = 2 + \frac{1}{a_n}$ for $n \geq 1$ with $a_1 = 3$.

7. Express $\displaystyle\sum_{k=3}^{5}(2k + 3)$ without sigma notation. You do not need to simplify.

8. Use sigma notation to write the sum $\frac{1}{3+4} + \frac{1}{4+5} + \frac{1}{5+6} + \cdots \frac{1}{9+10}$.

9. Suppose the sequence a_n is defined by $a_n = a_{n-1} + 2a_{n-2}$ with $a_0 = 1$ and $a_1 = 4$. Find a_2, a_3, and a_4.

10. Suppose $\displaystyle\sum_{n=1}^{20} x_n = 50$ and $\displaystyle\sum_{n=1}^{20}(x_n)^2 = 75$. Evaluate $\displaystyle\sum_{n=1}^{20}(2x_n + 1)^2$.

11. An arithmetic sequence has its firs term equal to 3 and its fourth term equal to 8. Find the twentieth term in the sequence.

12. An arithmetic sequence has first term 1 and common difference $\frac{3}{2}$. Find the sum of the first 15 terms of the sequence.

13. Evaluate $3 + 3^2 + 3^3 + 3^4$ by using the geometric sum formula. Show your work.

14. In a certain geometric sequence the first term is 2 and the third term is 5. If the common ratio is positive, find the second term in the sequence.

15. The first three terms of a geometric sequence are 2, $\frac{3}{2}$, $\frac{9}{8}$. Find the fifth term in the sequence.

16. Compute the sum of the infinite geometric series $1 + \frac{2}{5} + \left(\frac{2}{5}\right)^2 + \left(\frac{2}{5}\right)^3 + \cdots$.

17. An infinite geometric series has the first term equal to 0 and the sum equal to $\frac{7}{3}$. Find the ratio r.

18. Given the formula $1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$, evaluate $1^2 + 2^2 + 3^2 + \cdots + 10^2$.

19. Given that $1 + 3 + 5 + \cdots + (2n - 1) = n^2$, what is $1 + 3 + 5 + \cdots + (2n + 3)$?

20. Given the formula $1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$, evaluate $20 + 21 + 22 + \cdots + 40$.

21. Simplify $\frac{(n+1)!}{(n+3)!}$.

22. Let $a_1, a_2, a_3, \ldots,$ denote the sequence whose nth term is given by $a_n = \frac{n}{2+n}$. Compute $2a_3 + a_4$.

23. Evaluate the sum $\displaystyle\sum_{n=3}^{5}\frac{(-1)^n}{n}$.

24. Evaluate the sum $\displaystyle\sum_{n=1}^{5}\left(\frac{1}{n} - \frac{1}{n+1}\right)$.

25. Express $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \frac{1}{32} + \frac{1}{64}$ using sigma notation.

26. Suppose a sequence a_n is defined by $a_1 = 2$ and $a_n = na_{n-1}$, where $n \geq 2$. Determine a_4.

27. Find the common difference of the arithmetic sequence 18, $16\frac{1}{3}$, $14\frac{2}{3}$, 12, ...

28. Find the 48th term of the arithmetic sequence $-10, -7, -4, \ldots$.

29. Find the first three terms of an arithmetic sequence $a_1, a_2, a_3, \ldots$ such that $a_5 = 16$ and $a_{10} = 32$.

30. Find the sum of the first 20 terms of the arithmetic sequence defined by $a_n = 3 + 4(n-1)$.

31. Determine the common ratio of the geometric sequence 12, 8, $\frac{16}{3}$, $\frac{32}{9}$, ...

32. Find a formula for the sum $2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \cdots + \frac{2}{3^n}$.

33. Evaluate the infinite sum $2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \cdots + \frac{2}{3^n} + \cdots$.

34. Given the formulas $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$ and $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$, find a formula for $\sum_{k=1}^{n} k(k+1)$. Your answer should be a single fraction.

Chapter 8 Answers

1. $7\sqrt{2}$

2. Circle with radius 2, centered at the origin. Motion is clockwise.

3. $y = 4 + \left(\frac{x-1}{2}\right)^2$

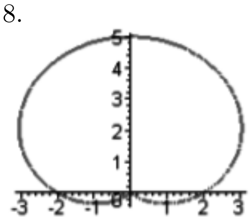
4. $\left(\frac{34}{9}, 0\right)$

5. $(-\sqrt{3}, -1)$

6. $\left(3\sqrt{2}, \frac{3\pi}{4}\right)$

7. $13 - 6\sqrt{2}$

8.



9. $\frac{1}{4}\tan\theta\sin\theta$; domain $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

10. $(x^2 + y^2)^2 = 2x^2 + 3y^2$

11. $A = 1$, $B = \frac{\ln 4}{\pi}$

12. Q is farthest from the origin

13. $(0, 5)$ and $(0, 1)$

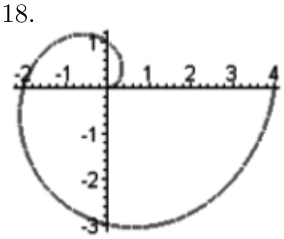
14. Circle centered at the origin with radius 2. Motion is clockwise.

15. $y = -\frac{2}{3}x + \frac{13}{3}$

16. $\left(2, \frac{\pi}{3}\right)$

17. $\left(2, \frac{\pi}{3}\right)$

18.



19. replacing θ by $-\theta$ does not leave the equation invariant (unchanged)

20. $x^2 = 4(y+1)$

21. $\sqrt{34}$

22. $56.31°$

23. $\langle -12, 13 \rangle$

24. $\langle \frac{3}{\sqrt{34}}, \frac{5}{\sqrt{34}} \rangle$

25. $\langle 0, 5\sqrt{10} \rangle$

26. $5\mathbf{i} + 2\mathbf{j}$

27. $\sqrt{29}$

28. $\overrightarrow{PR}$

29. 12.28

30. $\langle -5, 4 \rangle$

31. $\langle \frac{-5}{\sqrt{34}}, \frac{3}{\sqrt{34}} \rangle$

32. $\langle 7, 9 \rangle$

33. $\sqrt{85}$

34. 15.26

Chapter 11 Answers

1. 256

2. 195

3. $\frac{2^n - 1}{2^n}$

4. $4n^2 + 2n$

5. $\frac{43}{30}$

6. $\frac{17}{7}$

7. $9+11+13=33$

8. $\displaystyle\sum_{n=3}^{9}\frac{1}{2n+1}$

9. 6, 14, 26

10. 420

11. $\frac{104}{3}$

12. 195

13. 120

14. $\frac{\sqrt{10}}{2}$

15. $\frac{81}{128}$

16. $\frac{5}{3}$

17. $\frac{1}{7}$

18. $(n+1)^2$

19. 385

20. 630

21. n^2+5n+6

22. $\frac{28}{15}$

23. $-\frac{17}{60}$

24. $\frac{5}{6}$

25. $\displaystyle\sum_{n=0}^{6}\left(-\frac{1}{2}\right)^n$

26. 48

27. $1\frac{2}{3}=\frac{5}{3}$

28. 131

29. $\frac{16}{5}$, $\frac{32}{5}$, $\frac{48}{5}$

30. 820

31. $\frac{2}{3}$

32. $3\left(1-\left(\frac{1}{3}\right)^{n+1}\right)=3-\frac{1}{3^n}$

33. 3

34. $\frac{n(n+1)(n+2)}{3}$